# PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 70 NOVEMBER, 1944 NO. 9

## TECHNICAL PAPERS

AND

## DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

# STRESSES IN THE LININGS OF SHIELD-DRIVEN TUNNELS

BY ANDERS BULL,[1] M. AM. SOC. C. E.

SYNOPSIS

A rational method for determining the amounts and distribution of the soil reactions (passive pressures) developed around a shield-driven tunnel lining is presented in this paper.

Considering the lining supported by a hypothetical diaphragm, the thrust, shear, and moment produced at the support are first determined. From the values obtained, formulas are next derived, and tables prepared, permitting the deflections produced at various points of the lining under the combined influence of active loads and soil reactions to be expressed as simple product sums. However, the deflections may also be expressed in a different way, as functions of the soil reactions and the settlement. By equating the two sets of expressions a number of simultaneous equations are obtained from which the soil reactions are determined.

Applying the method to an actual case, the values obtained for the dimensions of the lining are compared with those obtained by the customary method of analysis, and the economic aspects thus raised are discussed, as are the comparative merits of steel as a lining material.

## PRESENT STATE OF ART

*1. Introductory Remarks.*—With few exceptions, shield-driven tunnels are of circular cross section. The shell or "lining" resisting the pressure of the surrounding ground is usually made of a succession of rings of iron, steel, or concrete, each of which is built up inside the shield from a number of segments. For purposes of design, the axial cross section through a ring at a joint is considered equivalent to one taken between joints as the latter is supposed to have the same strength as the segments proper. Although a tunnel lining is thus one

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 1, 1945.

[1] Civ. Engr., Board of Transportation, New York, N. Y.

of the simplest structures imaginable, its rational design is complicated by the highly indeterminate character of the pressure distribution.

Among the pressures acting on a lining may first be mentioned those due to the weight of the surrounding ground. They are referred to as "active pressures" and are determined with sufficient accuracy by means of certain broad rules roughly fitting the average soil condition anticipated within a given length of tunnel, it being obviously impracticable to make provision for minor variations. The accepted rule in the typical case of a sandy soil is to take the vertical pressure at a given point as the unit weight of the soil times the depth of the point below the surface, and the corresponding horizontal pressure as one third of that amount, additional allowance being made for the weight of the lining and the hydrostatic pressure of any water present.

Wherever the lining, as a result of these active pressures, is displaced in a direction away from the center of the tunnel, it causes the abutting ground to be compressed, and secondary pressures are developed which tend to resist the displacement. In sandy soils the latter pressures, according to the rules, are proportional to the amount of displacement, regardless of the depth of the point considered. They are generally referred to as "passive pressures." The word "passive" is misleading in this connection, however, because in a more widely used sense it denotes the maximum pressure that a soil is capable of exerting without being ruptured, a pressure which increases with the depth; so in order to preclude any confusion on this score, the term "soil reactions" will be used for pressures and forces due to displacement.

To narrow the scope of the paper, it will be assumed that the soil considered is of a type to which the foregoing rules may be applied.

*2. Original Theory.*—As stated, the active pressures exerted against a tunnel lining vary from point to point, depending on the depth below ground surface and water table, respectively. If the vertical pressures are plotted along a horizontal diameter, and vice versa, the pressure distribution is somewhat as indicated by the outlines of the unshaded diagrams in Fig. 1.

The thrust, shear, and moment which these pressures produce in the lining at some principal points along the periphery are found by treating the lining as an arch spanning a complete circle. In order to facilitate the mathematical work involved, the diagrams are divided into elementary shapes 1 to 11, Fig. 1, the effects due to each shape being determined separately, after which the combined effects are found by algebraic summation. The active pressures are: The downward pressure due to weight of soil (areas 1 and 2) and water (areas 3 and 4) above the horizontal diameter of the tunnel; the downward pressure due to weight of lining (area 5); the upward pressure due to hydrostatic action (areas 6 and 7); and the horizontal pressure due to soil (areas 8 and 9) and water (areas 10 and 11).

The horizontal pressures are usually symmetrical with respect to the vertical diameter of the lining, the corresponding forces being, therefore, in static equilibrium. Such is generally not the case with the active vertical pressures, the corresponding downward force being either larger or smaller than the upward one. If the downward force predominates (as assumed in this paper)

the lining will settle, compressing the ground below until soil reactions in the amount required for static equilibrium have been developed.

By the original theory,[2] the vertical soil reactions are assumed to be uniformly distributed over the horizontal diameter of the lining, as indicated by the shaded rectangle 12 in Fig. 1. By adding their effects to those produced by the

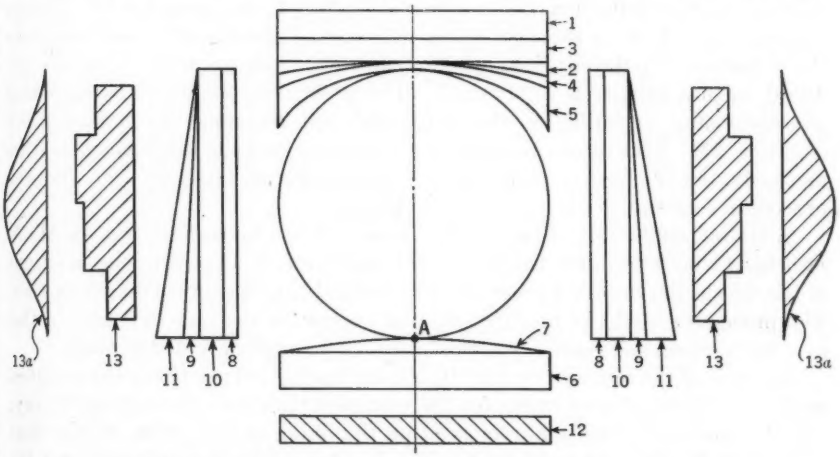


FIG. 1.—ASSUME SOIL PRESSURE DISTRIBUTION AROUND A LINING

active pressures 1 to 11, the total thrust, shear, and moment at each principal point are obtained, from which the required strength of the lining is determined.

It is clear at once that the theory disregards a most important feature—the deflection of the lining—since only for a perfectly rigid lining could the soil reactions be distributed in the manner claimed. Although the deflection may amount to only a few hundredths of an inch, the change it causes in the pressure distribution has a profound effect on the moments that to a large extent govern the stresses in the lining. This is due to the sensitive state of moment balance prevailing in a ring-shaped structure. As will be shown, the resultant moment at any point is the outcome of positive and negative moments whose magnitude may each be several hundred times that of the former, so that even a slight error in determining the component moments may upset the final result completely.

The failure to take the deflection into consideration leads to excessive values for the moments computed and thereby to designs which are very uneconomical. It seems unlikely, in fact, that a tunnel was ever built in strict compliance with the original theory, as a comparison with other tunnels, already long in service, would immediately have revealed that something must be radically wrong.

*3. Customary Method of Analysis.*—Not until the middle 1920's was serious attention paid to the relieving influence which the deflection has upon the moments in the lining. As a result of these investigations a theory was developed which may be considered the foundation of present-day tunnel design.

[2] "Shield and Compressed Air Tunneling," by B. H. M. Hewett and S. Johannesson, McGraw-Hill Book Co., Inc., New York, N. Y., 1922. p. 74.

The problem was not fully grasped at the outset, however, and the erroneous conception of a uniform distribution of the vertical soil reactions was thus inherited by the newer theory. As a novel feature, horizontal soil reactions were introduced which are subject to the following considerations:

Under the pressures 1 to 12, Fig. 1, the lining deflects, the horizontal components of the deflection being directed outward, compressing the ground horizontally. Due to the restraining influence of the resulting soil reactions the deflection is reduced; this, in turn, causes the soil reactions to be also reduced, until a balance is established. The problem is simplified by assuming a step-by-step variation of the horizontal soil reactions, as indicated by diagrams 13. The values required for a balance may be determined mathematically, but the further details are not open to discussion as no account of the procedure seems as yet to have been published.

A simple method for attaining the same end has been described by M. A. Drucker,[3] who determines the horizontal soil reactions as continuous functions of the depth, the resulting pressure distribution being shown by diagrams 13-a. The procedure would be an improvement except for the fact that also in this case the vertical soil reactions are assumed to be uniformly distributed.

Because of the restraint exerted by the horizontal soil reactions, the modified methods will yield lower values for the moments than does the original theory; but the obvious inconsistency of treating the lining as if it were, at the same time, both flexible and perfectly rigid makes it a foregone conclusion that the results obtained on such premises cannot be correct. The error incurred is found to increase with the flexibility of the lining.

In this paper the problem is approached from a different angle, permitting a method to be developed by which the complete pressure distribution due to a given loading condition may be determined in accordance with accepted mechanical laws and engineering practice. The important bearing which the flexibility of the lining has upon the economy of design—a feature which, so far, has been obscured because of the assumed fixed distribution of the vertical soil reactions—is thereby revealed.

*Notation.*—The letter symbols in this paper are defined where they first appear, either in the text or by diagram, and those used in solving specific problems are assembled for convenience of reference in the Appendix, where the adopted sign convention is also defined.

## Reactions in a Tunnel Lining

*4. Basic Principle.*—The thrust, shear, and moment produced at any point of a tunnel lining may be found by the same principle as the one on which the method of structural model analysis introduced by the late George E. Beggs,[4] M. Am. Soc. C. E., is based, and which may be briefly stated as follows: In an elastic structure, if a support is displaced by an infinitesimal amount in a given direction the resulting deflection line is identical with the influence line for the similarly directed reaction at that support.

[3] "Determination of Lateral Passive Soil Pressure and Its Effect on Tunnel Stresses," by M. A. Drucker, *Journal of the Franklin Institute*, May, 1943, p. 499.

[4] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

For a proof of the truth of the foregoing statement it is customary to refer to Maxwell's theorem, from which it was originally deduced. However, the same conclusion is reached more directly by considering that the work performed, respectively, by a load and the corresponding reaction during an infinitesimal displacement of the support must be of equal amount and opposite sign, since the work spent on the elastic deformation of the structure, being a function of the square of the displacement, becomes negligible. The latter method, which has the advantage of greater clarity, will be followed.

A circular tunnel ring lends itself particularly well to this type of analysis because of its assumedly uniform cross section and perfectly regular shape, as the result of which the deflection lines due to given displacements are of fixed shapes, comparable to the deflection lines for simple beams.

The ring in Fig. 2 is a 1-ft slice cut from a tunnel lining, the neutral circle of which is marked c. The ring is assumed to be cut at the invert point A and to be supported by a thin, inflexible diaphragm D, made up of two interlocking and detachable halves of which the upper half is rigidly connected to the left-hand end of the ring, while the lower half is rigidly connected to the right-hand end.

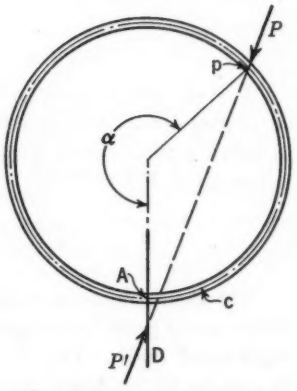


FIG. 2.—HYPOTHETICAL SUPPORT OF A TUNNEL RING

The reactions to be determined are those exerted by the diaphragm as a whole upon the right-hand end when a single force $P$ is applied to the ring at a point p, located on the neutral circle at an angular distance $\alpha$ from point A.

*5. Reactions at the Support.*—Referring to Fig. 3, the curved dashed lines represent the neutral circle of the ring, whereas the solid curves indicate the shapes assumed when the right-hand end is displaced in the various ways shown.

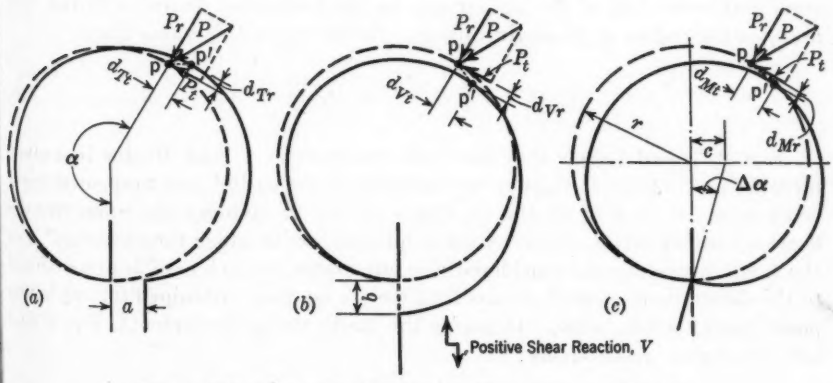


FIG. 3.—INFLUENCE LINES OBTAINED BY DISPLACING THE SUPPORT

For finding the thrust reaction $T$, the right-hand end is assumed to be displaced by an infinitesimal amount $a$ in the direction of positive thrust reaction (see Fig. 3(*a*)). The point p will then move to the new position p′. Resolving the deflection pp′ into its radial and tangential components $d_{Tr}$ and $d_{Tt}$, respectively, and the force $P$, similarly, into the components $P_r$ and $P_t$, one may write the work equation:

$$T\,a = P_r\,d_{Tr} + P_t\,d_{Tt} \quad \ldots\ldots (1)$$

a deflection component being considered positive when its direction is opposite to that of the force component by which it is multiplied (work negative). From Eq. 1 it follows that

$$T = P_r\frac{d_{Tr}}{a} + P_t\frac{d_{Tt}}{a} \quad \ldots\ldots (2)$$

For the shear reaction $V$, the right-hand end of the ring is assumed to be displaced downward by an amount $b$ in the direction of positive shear reaction. Resolving the deflection pp′ into its components $d_{Vr}$ and $d_{Vt}$, the work is expressed by: $V\,b = P_r\,d_{Vr} + P_t\,d_{Vt}$ or

$$V = P_r\frac{d_{Vr}}{b} + P_t\frac{d_{Vt}}{b} \quad \ldots\ldots (3)$$

For the moment reaction $M$, the right-hand end of the ring is rotated in the direction of positive moment reaction (so as to stretch the outer fibers in the right-hand end). This is done by tilting the lower half of the diaphragm clockwise by an angle $\Delta\alpha$ about the invert point. The deflection pp′ is resolved into the components $d_{Mr}$ and $d_{Mt}$, and the work equation becomes:

$$M\,\Delta\alpha = P_r\,d_{Mr} + P_t\,d_{Mt} \quad \ldots\ldots (4a)$$

The infinitesimal angle $\Delta\alpha$ is equal to $c/r$, in which $c$ is the intercept made by the produced lower half of the diaphragm on the horizontal diameter of the ring, and $r$ is the radius of its neutral fibers. Substituting this value gives:

$$M = r\left(P_r\frac{d_{Mr}}{c} + P_t\frac{d_{Mt}}{c}\right) \quad \ldots\ldots (4b)$$

Eqs. 2, 3, and 4 show that the three reactions $T$, $V$, and $M$, due to a given force $P$, are obtained, simply, by multiplying the radial and tangential force components $P_r$ and $P_t$ by the quotients formed in dividing the respective deflection components $d_r$ and $d_t$ by the displacements to which they are due. For the small displacements considered, the numerator, in each case, is proportional to the denominator, which means that the six quotients obtained for any given point p are fixed in value. Denoting the quotients by the letter $A$, Eqs. 2 to 4 take the forms, respectively:

$$T = P_r\,A_{Tr} + P_t\,A_{Tt} \quad \ldots\ldots (5a)$$

$$V = P_r\,A_{Vr} + P_t\,A_{Vt} \quad \ldots\ldots (5b)$$

$$M = r\,(P_r\,A_{Mr} + P_t\,A_{Mt}) \quad \ldots\ldots (5c)$$

the subscripts being self-explanatory. As the deflection pp′ changes from point to point along the periphery, both in amount and direction, the six factors $A$ (to be referred to henceforth as $A$-constants) will assume different values, each of them typical of the point considered.

When the ring is subjected to several forces, acting at different points, the thrust, shear, and moment reactions are determined separately for each force, and commensurable reactions are added up. The total reactions produced at the right-hand side of the support are, accordingly, expressed by the following general equations:

$$T = \sum(P_r A_{Tr}) + \sum(P_t A_{Tt}) \ldots\ldots (6a)$$

$$V = \sum(P_r A_{Vr}) + \sum(P_t A_{Vt}) \ldots\ldots (6b)$$

$$M = r\,[\sum(P_r A_{Mr}) + \sum(P_t A_{Mt})] \ldots\ldots (6c)$$

*6. Evaluating the A-Constants.*—In the case of a circular ring, formulas for the six $A$-constants at any point p may be derived once for all, the operation involving the following steps: (1) Determine the forces and moments required at the right-hand end of the ring to reproduce the displacements $a$, $b$, and $c$ ($\Delta\alpha$) in Fig. 3; (2) determine the radial and tangential deflections $d_r$ and $d_t$ simultaneously produced at any point by such forces and moments; and (3) divide the deflections by the corresponding displacements in accordance with Eqs. 2 to 4.

For the computations, which are omitted for the sake of brevity, the following standard deflection equations are used:

$$\Delta x = + \int \frac{M_o}{E\,I}\,(Y - y)\,ds \ldots\ldots (7a)$$

$$\Delta y = - \int \frac{M_o}{E\,I}\,(X - x)\,ds \ldots\ldots (7b)$$

$$\Delta\alpha = + \int \frac{M_o}{E\,I}\,ds \ldots\ldots (7c)$$

in which $X$ and $Y$ are the rectangular coordinates of point p, as referred to the vertical diameter of the ring and a tangent to the neutral circle at point A (positive to the right and upward, respectively); $x$ and $y$ are the coordinates of a moving point; $M_o$ = moment at point $x$, $y$; $s$ = length of arc along the neutral circle from point A to point $x$, $y$; $E$ = modulus of elasticity; and $I$ = moment of inertia of lining per foot of tunnel. The details of these computations are contained in the original complete manuscript, filed for reference in Engineering Societies Library.[4a]

The formulas thus derived are:

$$\left.\begin{aligned} A_{Tr} &= -\frac{\alpha \sin\alpha}{2\pi}; & A_{Tt} &= \mp\frac{\sin\alpha - \alpha\cos\alpha}{2\pi} \\ A_{Vr} &= -\frac{\sin\alpha - \alpha\cos\alpha}{2\pi}; & A_{Vt} &= \mp\frac{2\,(1 - \cos\alpha) - \alpha\sin\alpha}{2\pi} \\ A_{Mr} &= -\frac{\alpha\sin\alpha + 1 - \cos\alpha}{2\pi}; & A_{Mt} &= \mp\frac{\alpha\,(1 - \cos\alpha)}{2\pi} \end{aligned}\right\} \ldots (8)$$

[4a] 29 West 39th St., New York, N. Y.

In the right-hand column of Eqs. 8, the upper sign is used with angles $\alpha$ between 0 and $\pi$, the lower sign with angles between $\pi$ and $2\pi$.

*7. Load Concentrations.*—The loads on tunnel rings are usually distributed loads, as indicated in Fig. 1. By replacing these loads with equivalent load concentrations, acting at points spaced a uniform distance apart, any desired approximation to the true loading condition may be obtained by making the spacing close enough. A spacing equal to one sixteenth of the periphery will be suitable for present purposes.

Accordingly, the ring is considered composed of sixteen rigidly joined segments of equal size, with the respective load concentrations acting at the middle point of each segment. (These segments should not be confused with the segments into which a ring is divided for construction purposes.) For identifying the various segments, joints, middle points, forces, etc., the markings shown in Fig. 4 are used. Subscripts are used when necessary for specifying the location or character with which a quantity is identified. Thus, $A_{M5}$ would denote the $A$-constant by which the tangential component of a force at point 5 is multiplied in determining its share of the moment reaction at the support, whereas $A_5$ would denote any or all of the six $A$-constants typical of point 5.

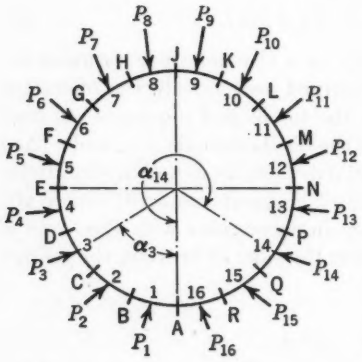


FIG. 4.—IDENTIFICATION MARKINGS

The location of the points to be considered having thus been fixed, the numerical values of the corresponding $A$-constants are next computed from Eqs. 8, and arranged, as shown in Table 1.

TABLE 1.—$A$-CONSTANTS FOR THRUST, SHEAR, AND MOMENT REACTIONS AT THE INVERT JOINT

| Force | $A_{Tr}$ | $A_{Vr}$ | $A_{Mr}$ | Force | $A_{Tl}$ | $A_{Vl}$ | $A_{Ml}$ |
|---|---|---|---|---|---|---|---|
| $P_{r1}$ | −0.00610 | −0.00040 | −0.00915 | $P_{l1}$ | −0.00040 | −0.00002 | −0.00060 |
| | .... | .... | −0.00822 | | .... | .... | .... |
| $P_{r2}$ | −0.05208 | −0.01047 | −0.07891 | $P_{l2}$ | −0.01047 | −0.00156 | −0.01580 |
| $P_{r3}$ | −0.12992 | −0.04552 | −0.20065 | $P_{l3}$ | −0.04552 | −0.01155 | −0.06944 |
| $P_{r4}$ | −0.21455 | −0.11342 | −0.34265 | $P_{l4}$ | −0.11342 | −0.04166 | −0.17607 |
| $P_{r5}$ | −0.27585 | −0.21097 | −0.46605 | $P_{l5}$ | −0.21097 | −0.10456 | −0.33612 |
| $P_{r6}$ | −0.28582 | −0.32331 | −0.53339 | $P_{l6}$ | −0.32331 | −0.20934 | −0.53473 |
| $P_{r7}$ | −0.22570 | −0.42621 | −0.51719 | $P_{l7}$ | −0.42621 | −0.35727 | −0.74403 |
| $P_{r8}$ | −0.09145 | −0.49079 | −0.40670 | $P_{l8}$ | −0.49079 | −0.53905 | −0.92849 |
| $P_{r9}$ | +0.10364 | −0.48999 | −0.21161 | $P_{l9}$ | +0.48999 | +0.73415 | +1.05229 |
| $P_{r10}$ | +0.32987 | −0.40526 | +0.03838 | $P_{l10}$ | +0.40526 | +0.91284 | +1.08744 |
| $P_{r11}$ | +0.54565 | −0.23226 | +0.29808 | $P_{l11}$ | +0.23226 | +1.04081 | +1.02084 |
| $P_{r12}$ | +0.70494 | +0.01588 | +0.51473 | $P_{l12}$ | −0.01588 | +1.08535 | +0.85897 |
| $P_{r13}$ | +0.76624 | +0.30851 | +0.63813 | $P_{l13}$ | −0.30851 | +1.02245 | +0.62884 |
| $P_{r14}$ | +0.70155 | +0.60109 | +0.63082 | $P_{l14}$ | −0.60109 | +0.84302 | +0.37499 |
| $P_{r15}$ | +0.50349 | +0.84194 | +0.47666 | $P_{l15}$ | −0.84194 | +0.55713 | +0.15273 |
| $P_{r16}$ | +0.18899 | +0.98119 | +0.18594 | $P_{l16}$ | −0.98119 | +0.19511 | +0.01861 |
| | .... | .... | −0.00822 | | | | |

*8. Adjusting for Polygonal Effect.*—Because of the delicate balance of positive and negative moments in a tunnel ring, the substitution of forces for

distributed loads may give rise to an appreciable error in the value obtained for the moment reaction $M$ in Eqs. 6, making an adjustment necessary. For an illustration, assume that a ring is subjected to a radial pressure of a uniform intensity along its entire periphery. The pressure line then takes the shape of a circle coinciding with the neutral fibers, as indicated in Fig. 5. However, if the uniform pressure is replaced by sixteen equivalent forces $P_r$, acting at the middle points of the segments, the pressure line changes into a polygon, a moment being produced at each of the sixteen joints, amounting to

$$M = + 0.01644\, P_r\, r \ldots\ldots (9)$$

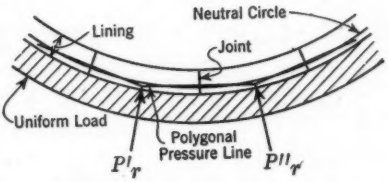


FIG. 5.—PRESSURE LINE AS AFFECTED BY TYPE OF LOADING

in which $r$ is the radius of the neutral circle. To compensate for this moment, which is essentially a local effect due to the two forces $P'_r$ and $P''_r$ straddling the joint, a negative moment of equal amount must be added. For the invert joint, $P'_r$ and $P''_r$ become $P_{r1}$ and $P_{r16}$, respectively; therefore the moment to be added to the product sum $\sum(P_r\, A_{Mr})$ in Eqs. 6 is:

$$-M = -0.01644\,\frac{P'_r + P''_r}{2} = -0.00822\,(P_{r1} + P_{r16}) \ldots\ldots (10)$$

as indicated in Table 1. The adjustment does not pretend to be accurate when a load distribution is not uniform, but it meets all practical requirements and is easy to apply. No great error results from the concentration of tangential pressures; nor is any adjustment required in the thrust or shear reaction.

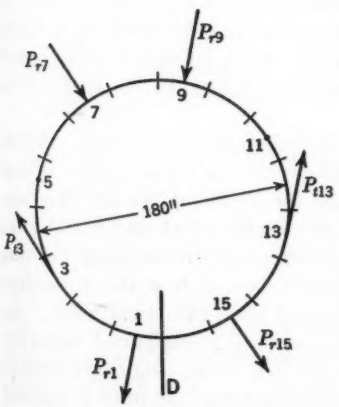


FIG. 6.—LOADING OF THE RING

*9. Example.*—A 180-in. ring (Fig. 6) is supported by the diaphragm D and subjected to forces $P_1$ to $P_{15}$, Table 2. The radial forces are concentrations of distributed loads on the respective segments. The problem is to determine the resulting reactions at the right-hand side of the support. The forces, multiplied by the corresponding $A$-constants in Table 1, yield the products entered under the respective headings. The products are added algebraically, as in Eqs. 6, and the moment product sum is multiplied by the radius: $T = -1.08317 - 1.77015 = -2.85332$ kips; $V = -4.01595 + 5.05450 = +1.03855$ kips; and $M = \frac{180}{2}(-3.14551 + 2.79700) = -31.3659$ in-kips.

*10. Forces in Static Equilibrium.*—Along with the reactions $T$, $V$, and $M$, conjugate reactions $T'$, $V'$, and $M'$ are set up simultaneously at the left-hand

side of the diaphragm D. With only a single force $P$ applied to the ring, the sum of each pair of conjugate reactions represents, respectively, the total thrust, shear, and moment that must be exerted on the ring by the support as a whole in order that static equilibrium be maintained. In order to be able to exert

TABLE 2.—DETERMINING THE REACTIONS FOR A SPECIFIED LOADING

| Force | | Reactions Caused by Radial Forces | | | Reactions Caused by Tangential Forces | | |
|---|---|---|---|---|---|---|---|
| Fig. 6 | Kips | $P_r A_{T_r}$ | $P_r A_{V_r}$ | $P_r A_{M_r}$ | $P_t A_{T_t}$ | $P_t A_{V_t}$ | $P_t A_{M_t}$ |
| $P_{r1}$ | −2 | +0.01220 | +0.00080 | +0.01830 | .... | .... | .... |
| $P_{t3}$ | +5 | .... | .... | +0.01644 | .... | .... | .... |
| $P_{r7}$ | +4 | −0.90280 | −1.70484 | −2.06876 | −0.22760 | −0.05775 | −0.34720 |
| $P_{r9}$ | +3 | +0.31092 | −1.46997 | −0.63483 | .... | .... | .... |
| $P_{t13}$ | +5 | .... | .... | .... | .... | .... | .... |
| $P_{r15}$ | −1 | −0.50349 | −0.84194 | −0.47666 | −1.54255 | +5.11225 | +3.14420 |
| Total | .... | −1.08317 | −4.01595 | −3.14551 | −1.77015 | +5.05450 | +2.79700 |

those reactions, the support must be restrained by subjecting it to an outside force $P'$, equal and opposite to the force $P$, as shown in Fig. 2.

However, if the ring is subjected to a system of forces which are by themselves in static equilibrium, it is obvious that there can be no reaction left for the support to exert, in which case the support can have no influence on the stresses in the ring.

The forces acting on a tunnel ring must always be in static equilibrium as the ring would otherwise be subject to accelerating motion. The three reactions found from Eqs. 6 are then simply the reactions exerted by the ring portion to the left of the invert point upon the portion immediately to the right of it, being numerically equal to the thrust, shear, and moment produced in the ring at that point.

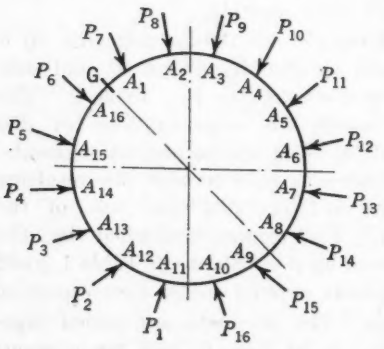


FIG. 7.—SHIFTING OF *A*-CONSTANTS

*11. Reactions at Other Joints.*—When the reactions at the invert joint, as well as all the forces acting on the ring, are known, the reactions at any other joint may be determined by treating the ring as a simple beam; but the procedure involves a lot of arithmetical work. An easier method will be explained by an example. In Fig. 7, the ring is assumed to be cut and supported at joint G instead of at the invert joint. Turning the diagram until joint G is at the bottom, one may imagine that the ring end to the right is displaced in the same manner and by the same amounts as was the right-hand end in Fig. 3.

It is evident that the resulting deflection lines will be identical with those obtained in the former case, yielding the same values for the $A$-constants, except that the latter are now referred to joint G. The effect of this procedure is that

the constants are shifted with respect to the forces, as shown in Fig. 7. However, since the tangential components of the forces are considered positive when directed upward—that is, away from the invert joint—the trend of the components $P_{t1}$ to $P_{t6}$ and $P_{t9}$ to $P_{t14}$ with respect to joint G is reversed. This has the same effect on the products $P_t A_t$ as if the signs of the constants $A_t$ for points 1 to 6, and 9 to 14, were reversed. Thus, for the moment reaction at joint G,

$$M_G = r\,[P_{r1}\,0.29808 + P_{r2}\,0.51473 \cdots - P_{r15}\,0.21161 + P_{r16}\,0.03838 - (P_{r6} + P_{r7})\,0.00822 - P_{t1}\,1.02084 - P_{t2}\,0.85897 \cdots - P_{t6}\,0.01861 - P_{t7}\,0.00060 - P_{t8}\,0.01580 + P_{t9}\,0.06944 + P_{t10}\,0.17607 \cdots + P_{t14}\,0.92849 + P_{t15}\,1.05229 + P_{t16}\,1.08744] \ldots (11)$$

The reactions found are those exerted by the support on the ring portion extending counterclockwise from joint G. When the forces are in static equilibrium, the reactions are numerically equal to the thrust, shear, and moment in the ring at that joint, for the reason explained in Section 10.

*12. Special Case of Symmetrical Loading.*—The loading on tunnel linings is mostly symmetrical with respect to the vertical diameter, which means that $P_1 = P_{16}$; $P_2 = P_{15}$; $\cdots P_8 = P_9$. For use in such cases, Table 1 may be put into a more convenient form by combining the $A$-constants, by which the radial and tangential components of any two equal forces would have to be multiplied, into a single constant, thereby reducing the number of multiplications. The factor $-0.00822$ in Eq. 10 is likewise included in the new constants, which will be referred to as $B$-constants and identified by the letter $B$. The $B_M$-constants for the invert joint, for example, are computed as shown in Table 3.

TABLE 3.—COMPUTATION OF $B_M$-CONSTANTS FOR INVERT JOINT A

| Constant | For radial forces | For tangential forces |
|---|---|---|
| $B_{M1}$ | $-0.00915 + 0.18594 - 2 \times 0.00822 = +0.16035$ | $-0.00060 + 0.01861 = +0.01801$ |
| $B_{M2}$ | $-0.07891 + 0.47666 = +0.39775$ | $-0.01580 + 0.15273 = +0.13693$ |
| .... | .... | .... |
| .... | .... | .... |
| .... | .... | .... |
| $B_{M8}$ | $-0.40670 - 0.21161 = -0.61831$ | $-0.92849 + 1.05229 = +0.12380$ |

The $B$-constants for any other joint are obtained by the same process of shifting as was described in Section 11, due attention being paid to the signs of tangential components. Again taking joint G for an example (see Fig. 7), the combination of the $A_M$-constants for equal forces is demonstrated by Table 4.

TABLE 4.—COMPUTATION OF $B_M$-CONSTANTS FOR JOINT G

| Constant | Radial constants, $B_r$ | Tangential constants, $B_t$ |
|---|---|---|
| $B_{M1}$ | $+A_{Mr11} + A_{Mr10} = +0.29808 + 0.03838 = +0.33646$ | $-A_{Mt11} + A_{Mt10} = -1.02084 + 1.08744 = +0.06660$ |
| .... | .... | .... |
| .... | .... | .... |
| .... | .... | .... |
| $B_{M6}$ | $+0.18594 - 0.46605 - 0.00822 = -0.28833$ | $-0.01861 + 0.33612 = +0.31751$ |
| $B_{M7}$ | $-0.00915 - 0.34265 - 0.00822 = -0.36002$ | $-0.00060 + 0.17607 = +0.17547$ |
| $B_{M8}$ | $-0.07891 - 0.20065 = -0.27956$ | $-0.01580 + 0.06944 = +0.05364$ |

The $B$-constants for moment and thrust reactions at the joints $A$ to $J$ are arranged in Table 5, permitting the reactions at any joint to be determined with equal ease, it being noted that the reactions are alike at symmetrical joints, such as $B$ and $R$ in Fig. 4. No $B$-constants are given for shear reactions; in the

TABLE 5.—$B$-CONSTANTS FOR USE WITH SYMMETRICAL LOADS

| Force | Joint (See Fig. 4) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | J |
| (*a*) For Moment Reactions Caused by Radial Components | | | | | | | | | |
| $P_{r1}$ | +0.1603 | +0.6544 | +1.1075 | +1.2690 | +1.1529 | +0.8128 | +0.3365 | −0.1732 | −0.6183 |
| $P_{r2}$ | +0.3978 | +0.6134 | +0.8158 | +0.9914 | +0.9289 | +0.6765 | +0.3031 | −0.1086 | −0.4788 |
| $P_{r3}$ | +0.4302 | +0.5592 | +0.4974 | +0.4758 | +0.5150 | +0.4192 | +0.2314 | −0.0025 | −0.2353 |
| $P_{r4}$ | +0.2955 | +0.3141 | +0.2192 | +0.0210 | −0.0339 | +0.0700 | +0.1136 | +0.1047 | +0.0487 |
| $P_{r5}$ | +0.0487 | −0.0446 | −0.1603 | −0.2905 | −0.4241 | −0.3395 | −0.0567 | +0.1648 | +0.2955 |
| $P_{r6}$ | −0.2353 | −0.4277 | −0.5543 | −0.6074 | −0.5961 | −0.5508 | −0.2883 | +0.1340 | +0.4302 |
| $P_{r7}$ | −0.4788 | −0.7450 | −0.8728 | −0.8598 | −0.7340 | −0.5450 | −0.3600 | −0.0229 | +0.3978 |
| $P_{r8}$ | −0.6183 | −0.9239 | −1.0506 | −0.9994 | −0.8087 | −0.5433 | −0.2796 | −0.0963 | +0.1603 |
| (*b*) For Moment Reactions Caused by Tangential Components | | | | | | | | | |
| $P_{t1}$ | +0.0180 | +0.1341 | +0.2223 | +0.2538 | +0.2301 | +0.1619 | +0.0666 | −0.0351 | −0.1238 |
| $P_{t2}$ | +0.1369 | +0.3744 | +0.6102 | +0.7062 | +0.6459 | +0.4586 | +0.1933 | −0.0924 | −0.3434 |
| $P_{t3}$ | +0.3055 | +0.6130 | +0.8584 | +1.0022 | +0.9347 | +0.6773 | +0.2997 | −0.1149 | −0.4861 |
| $P_{t4}$ | +0.4528 | +0.7895 | +1.0050 | +1.0868 | +1.0337 | +0.7758 | +0.3690 | −0.0941 | −0.5229 |
| $P_{t5}$ | +0.5229 | +0.8448 | +1.0180 | +1.0365 | +0.9279 | +0.7254 | +0.3820 | −0.0389 | −0.4528 |
| $P_{t6}$ | +0.4861 | +0.7513 | +0.8762 | +0.8591 | +0.7282 | +0.5341 | +0.3175 | +0.0233 | −0.3055 |
| $P_{t7}$ | +0.3434 | +0.5176 | +0.5924 | +0.5680 | +0.4653 | +0.3203 | +0.1755 | +0.0508 | −0.1369 |
| $P_{t8}$ | +0.1238 | +0.1845 | +0.2093 | +0.1986 | +0.1600 | +0.1066 | +0.0536 | +0.0152 | −0.0180 |
| (*c*) For Thrust Reactions Caused by Radial Components | | | | | | | | | |
| $P_{r1}$ | +0.1829 | +0.6925 | +1.2050 | +1.4678 | +1.4712 | +1.2506 | +0.8755 | +0.4335 | +0.0122 |
| $P_{r2}$ | +0.4514 | +0.6955 | +0.9552 | +1.2084 | +1.2472 | +1.0961 | +0.8086 | +0.4542 | +0.1042 |
| $P_{r3}$ | +0.5716 | +0.7142 | +0.6988 | +0.7346 | +0.8334 | +0.8052 | +0.6748 | +0.4792 | +0.2598 |
| $P_{r4}$ | +0.5517 | +0.5750 | +0.4936 | +0.3238 | +0.2926 | +0.4120 | +0.4759 | +0.4804 | +0.4291 |
| $P_{r5}$ | +0.4291 | +0.3311 | +0.2000 | +0.0516 | −0.0975 | −0.0367 | +0.2177 | +0.4257 | +0.5517 |
| $P_{r6}$ | +0.2598 | +0.0540 | −0.1109 | −0.2214 | −0.2778 | −0.2919 | −0.0869 | +0.2889 | +0.5716 |
| $P_{r7}$ | +0.1042 | −0.1822 | −0.3673 | −0.4402 | −0.4157 | −0.3279 | −0.2206 | +0.0591 | +0.4514 |
| $P_{r8}$ | +0.0122 | −0.3171 | −0.5115 | −0.5617 | −0.4904 | −0.3445 | −0.1820 | −0.0582 | +0.1829 |
| (*d*) For Thrust Reactions Caused by Tangential Components | | | | | | | | | |
| $P_{t1}$ | −0.9816 | +0.1392 | +0.2408 | +0.2926 | +0.2926 | +0.2481 | +0.1730 | +0.0847 | +0.0008 |
| $P_{t2}$ | −0.8524 | −0.6015 | +0.6727 | +0.8261 | +0.8334 | +0.7138 | +0.5059 | +0.2585 | +0.0209 |
| $P_{t3}$ | −0.6466 | −0.3190 | −0.0163 | +1.2134 | +1.2472 | +1.0911 | +0.7993 | +0.4421 | +0.0910 |
| $P_{t4}$ | −0.4219 | −0.0614 | +0.2218 | +0.4049 | +1.4712 | +1.3327 | +1.0273 | +0.6318 | +0.2268 |
| $P_{t5}$ | −0.2268 | +0.1188 | +0.3597 | +0.4795 | +0.4904 | +1.4074 | +1.1652 | +0.8121 | +0.4219 |
| $P_{t6}$ | −0.0910 | +0.1943 | +0.3766 | +0.4453 | +0.4157 | +0.3229 | +1.1922 | +0.9554 | +0.6466 |
| $P_{t7}$ | −0.0209 | +0.1667 | +0.2798 | +0.3128 | +0.2778 | +0.2005 | +0.1130 | +1.0267 | +0.8524 |
| $P_{t8}$ | −0.0008 | +0.0646 | +0.1029 | +0.1123 | +0.0975 | +0.0679 | +0.0351 | +0.0101 | +0.9816 |

rare cases where they are of interest, the loading is usually asymmetrical, and Table 1 must be used.

Table 5 is used in the same way as Table 1: Each force component acting on the left half of the ring is multiplied by the $B$-constant given in the same line under the proper heading, and the products are added. The moment reaction at any joint may thus be expressed symbolically by the formula:

$$M_N = r\,[\Sigma(P_r\,B_N) + \Sigma(P_t\,B_N)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which the subscript $N$ stands for any of the letters A to J, identifying the joint, and $B_N$ stands for the corresponding sixteen $B$-constants in Tables 5($a$) and 5($b$).

## Deflection of a Tunnel Lining

*13.* The next step involves the evaluation of the radial deflection $d$ (Fig. 8) produced at any point q of a ring by a single force $P$ applied at a point p and having the radial and tangential components $P_r$ and $P_t$, respectively. This deflection is in no way related to the deflections $d_{Tr}$ to $d_{Mt}$ referred to in discussing Fig. 3, which are due to the displacement of one end of the cut ring at the support and which are independent of the stiffness of the ring.

Formulas for $d$ may be derived by means of Eqs. 7, expressions being first obtained for the rectangular deflections $\Delta x$ and $\Delta y$. The calculations (on file in Engineering Societies Library[4a]) yield the following equations:

$$d = \frac{r^3}{EI}\left\{ T\left(1 - \cos\beta - \frac{\beta \sin\beta}{2}\right) + V\left(\frac{\beta\cos\beta}{2} - \frac{\sin\beta}{2}\right) + \frac{M}{r}(\cos\beta - 1) + P_r\left[\frac{\cos\alpha \sin\beta}{2} - \frac{\beta\cos(\alpha - \beta)}{2}\right] \pm P_t\left[\frac{\sin\alpha\sin\beta}{2} - \frac{\beta\sin(\alpha-\beta)}{2} + \cos\beta - 1\right]\right\} \quad \ldots\ldots (13a)$$

and

$$d = \frac{r^3}{EI}\left\{ T\left(1 - \cos\beta - \frac{\beta \sin\beta}{2}\right) + V\left(\frac{\beta\cos\beta}{2} - \frac{\sin\beta}{2}\right) + \frac{M}{r}(\cos\beta - 1) + P_r\left[\frac{\sin\alpha\cos\beta}{2} - \frac{\alpha\cos(\beta-\alpha)}{2}\right] \pm P_t\left[\cos\beta - \cos(\beta-\alpha) + \frac{\sin\alpha\sin\beta}{2} + \frac{\alpha\sin(\beta-\alpha)}{2}\right]\right\} \quad \ldots (13b)$$

The upper sign is used with $P_t$ for values of $\alpha$ between 0 and $\pi$, the lower sign for values between $\pi$ and $2\pi$. Eq. 13$a$ covers the ring portion extending clockwise from the invert joint to point p, and Eq. 13$b$ covers the remainder of the ring. In order not to make the formulas too bulky, the deflection is expressed as a function of the reactions at the invert joint.

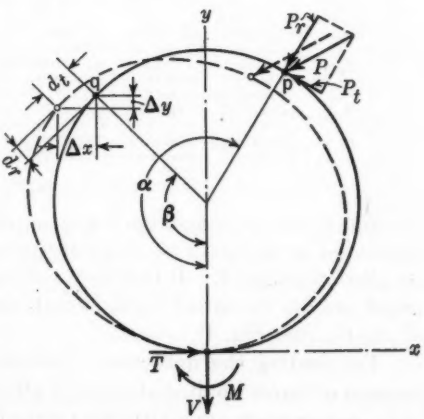


Fig. 8.—Ring Deflected by a Single Force

*14. C-Constants.*—With the aid of tables, the deflection $d$ may be determined with the same ease as are the reactions $T$, $V$, and $M$, by multiplying the components $P_r$ and $P_t$ by a set of constants, which are, in this case, functions of two angles—$\alpha$, which locates point p at which the force is applied; and $\beta$, which locates point q

for which the deflection is sought. The new constants will be named *C*-constants. Their numerical values for all possible combinations of the sixteen values of $\alpha$ and of $\beta$ identified with points 1 to 16 are given in Table 6. Because of symmetry, the deflections produced at points 9 to 16 by

TABLE 6.—*C*-CONSTANTS FOR RADIAL DEFLECTIONS

(Use Forces in Parentheses for Determining Deflection at Points (9) to (16))

| Force | POINT (SEE FIG. 4) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 (16) | 2 (15) | 3 (14) | 4 (13) | 5 (12) | 6 (11) | 7 (10) | 8 (9) |
| (*a*) *C*-CONSTANTS FOR RADIAL COMPONENTS $P_r$ | | | | | | | | |
| $P_{r1}$ ($P_{r16}$) | −0.002328 | −0.008070 | −0.011322 | −0.011827 | −0.009827 | −0.005986 | −0.001226 | +0.003457 |
| $P_{r2}$ ($P_{r15}$) | −0.008070 | −0.050167 | −0.083099 | −0.093640 | −0.082771 | −0.055201 | −0.018184 | +0.020051 |
| $P_{r3}$ ($P_{r14}$) | −0.011322 | −0.083099 | −0.168932 | −0.211681 | −0.200916 | −0.145894 | −0.063050 | +0.027728 |
| $P_{r4}$ ($P_{r13}$) | −0.011827 | −0.093640 | −0.211681 | −0.303417 | −0.316388 | −0.251262 | −0.132122 | +0.009298 |
| $P_{r5}$ ($P_{r12}$) | −0.009827 | −0.082771 | −0.200916 | −0.316388 | −0.373064 | −0.330918 | −0.206106 | −0.037813 |
| $P_{r6}$ ($P_{r11}$) | −0.005986 | −0.055201 | −0.145894 | −0.251262 | −0.330918 | −0.340739 | −0.254397 | −0.100355 |
| $P_{r7}$ ($P_{r10}$) | −0.001226 | −0.018184 | −0.063050 | −0.132122 | −0.206106 | −0.254397 | −0.242681 | −0.152198 |
| $P_{r8}$ ($P_{r9}$) | +0.003457 | +0.020051 | +0.027728 | +0.009298 | −0.037813 | −0.100355 | −0.152198 | −0.160835 |
| $P_{r9}$ ($P_{r8}$) | +0.007192 | +0.051947 | +0.107239 | +0.140507 | +0.130721 | +0.074008 | −0.014939 | −0.106020 |
| $P_{r10}$ ($P_{r7}$) | +0.009388 | +0.072035 | +0.160709 | +0.234881 | +0.261469 | +0.223190 | +0.123209 | −0.014939 |
| $P_{r11}$ ($P_{r6}$) | +0.009825 | +0.077856 | +0.180446 | +0.276721 | +0.329285 | +0.313377 | +0.223190 | +0.074008 |
| $P_{r12}$ ($P_{r5}$) | +0.008673 | +0.070251 | +0.166946 | +0.263851 | +0.326537 | +0.329285 | +0.261469 | +0.130721 |
| $P_{r13}$ ($P_{r4}$) | +0.006428 | +0.052964 | +0.128279 | +0.207307 | +0.263851 | +0.276721 | +0.234881 | +0.140507 |
| $P_{r14}$ ($P_{r3}$) | +0.003790 | +0.031681 | +0.077950 | +0.128279 | +0.166946 | +0.180446 | +0.160709 | +0.107239 |
| $P_{r15}$ ($P_{r2}$) | +0.001500 | +0.012698 | +0.031681 | +0.052964 | +0.070251 | +0.077856 | +0.072035 | +0.051947 |
| $P_{r16}$ ($P_{r1}$) | +0.000175 | +0.001500 | +0.003790 | +0.006428 | +0.008673 | +0.009825 | +0.009388 | +0.007192 |
| (*b*) *C*-CONSTANTS FOR TANGENTIAL COMPONENTS $P_t$ | | | | | | | | |
| $P_{t1}$ ($P_{t16}$) | −0.000173 | −0.000547 | −0.000756 | −0.000783 | −0.000646 | −0.000389 | −0.000073 | +0.000236 |
| $P_{t2}$ ($P_{t15}$) | −0.002287 | −0.011539 | −0.017765 | −0.019406 | −0.016745 | −0.010801 | −0.003080 | +0.004743 |
| $P_{t3}$ ($P_{t14}$) | −0.006183 | −0.038410 | −0.067897 | −0.079142 | −0.071661 | −0.049261 | −0.018078 | +0.014774 |
| $P_{t4}$ ($P_{t13}$) | −0.010817 | −0.073854 | −0.144437 | −0.182216 | −0.174231 | −0.127367 | −0.055838 | +0.023037 |
| $P_{t5}$ ($P_{t12}$) | −0.015142 | −0.109136 | −0.227102 | −0.306739 | −0.312428 | −0.243253 | −0.122540 | +0.018251 |
| $P_{t6}$ ($P_{t11}$) | −0.018293 | −0.136665 | −0.296410 | −0.420424 | −0.453890 | −0.378094 | −0.214367 | −0.008739 |
| $P_{t7}$ ($P_{t10}$) | −0.019723 | −0.151250 | −0.338029 | −0.496975 | −0.561451 | −0.497878 | −0.314464 | −0.059196 |
| $P_{t8}$ ($P_{t9}$) | −0.019267 | −0.150790 | −0.344894 | −0.521272 | −0.610055 | −0.568995 | −0.394272 | −0.122610 |
| $P_{t9}$ ($P_{t8}$) | +0.017133 | +0.136340 | +0.317747 | +0.491036 | +0.591141 | +0.574045 | +0.427859 | +0.176840 |
| $P_{t10}$ ($P_{t7}$) | +0.013822 | +0.111550 | +0.264109 | +0.415789 | +0.512391 | +0.514218 | +0.405920 | +0.201125 |
| $P_{t11}$ ($P_{t6}$) | +0.009992 | +0.081647 | +0.195979 | +0.313498 | +0.394100 | +0.406558 | +0.336157 | +0.188889 |
| $P_{t12}$ ($P_{t5}$) | +0.006315 | +0.052173 | +0.126772 | +0.205674 | +0.263090 | +0.277910 | +0.238795 | +0.147288 |
| $P_{t13}$ ($P_{t4}$) | +0.003325 | +0.027747 | +0.068179 | +0.112033 | +0.145553 | +0.156986 | +0.139383 | +0.092448 |
| $P_{t14}$ ($P_{t3}$) | +0.001317 | +0.011100 | +0.027559 | +0.045821 | +0.060381 | +0.066352 | +0.060619 | +0.042647 |
| $P_{t15}$ ($P_{t2}$) | +0.000301 | +0.002559 | +0.006419 | +0.010791 | +0.014408 | +0.016106 | +0.015094 | +0.011157 |
| $P_{t16}$ ($P_{t1}$) | +0.000012 | +0.000098 | +0.000250 | +0.000425 | +0.000573 | +0.000651 | +0.000625 | +0.000482 |

a unit radial or tangential force applied, say, at point 3, must equal those produced at points 8 to 1, respectively, by a unit radial or tangential force applied at point 14. It therefore suffices to register the *C*-constants for the first eight points, the forces being simply used in reversed order with the constants $C_9$ to $C_{16}$ (see Fig. 4).

For finding the deflection $d$ produced at any of the sixteen points by a system of forces applied at some or all of the same sixteen points, the respective force components are multiplied by the *C*-constants in the corresponding column and lines, after which the products are added and their sum is multiplied by the factor $\frac{r^3}{E\,I}$. It should be noted that the deflection thus found is the one pro-

duced in a ring supported as shown in Fig. 2. Only when the forces are in static equilibrium is the restraining influence of the support eliminated, as was explained in Section 10.

*15. Example. To Determine the Amount by Which the Length of a Diameter Through Points 3 and 11 on the Ring in Fig. 6 Has Been Changed as a Result of Radial Deflections Due to the Forces Shown.*—Assuming $E = 30{,}000$ kips per sq in. and $I = 4$ in.$^4$, then $\frac{r^3}{E\,I} = \frac{729{,}000}{120{,}000} = 6.075$. The computations, given in Table 7, show that the deflections at points 3 and 11 are both positive. Therefore, the diameter is lengthened by the amount $+ 0.376 + 1.957 = 2.333$ in. The lengthening would be the same if the ring were not supported, the forces in the example being in static equilibrium.

TABLE 7.—COMPUTATION OF CHANGE IN DIAMETER CAUSED BY RADIAL DEFLECTIONS

| Force | | $P\,C_3$ | $P\,C_{11}$ |
|---|---|---|---|
| Fig. 6 | Kips | | |
| Radial Force $P_r$— | | | |
| $P_{r1}$ | −2 | +0.022644 | −0.019650 |
| $P_{r7}$ | +4 | −0.252200 | +0.892760 |
| $P_{r9}$ | +3 | +0.321717 | −0.301065 |
| $P_{r15}$ | −1 | −0.031681 | +0.055201 |
| Tangential Force $P_t$— | | | |
| $P_{t3}$ | +5 | −0.339485 | +0.331760 |
| $P_{t13}$ | +5 | +0.340895 | −0.636835 |
| Total | .... | +0.061890 | +0.322171 |
| Total times 6.075 | .... | +0.376 | +1.957 |

*16. D-Constants.*— In the most common case of a symmetrically loaded tunnel lining, Table 6 may be simplified by combining, in each column, the two constants used with symmetrical and equal forces, just as was done in deriving the *B*-constants. The constants thus obtained will be referred to as *D*-constants. Their numerical values are given in Table 8, which is used in the same manner as Table 6. For symmetrical loading, the radial deflection $d$ at any of the eight points in the left half of the ring may be expressed by the formula:

$$d_n = \frac{r^3}{E\,I}\,[\Sigma(P_r\,D_n) + \Sigma(P_t\,D_n)] \qquad (14)$$

in which the subscript $n$ refers to any of the numbers 1 to 8 designating the point, and $D_n$ stands for the sixteen *D*-constants given for that point in Table 8.

## SOIL REACTIONS

*17. Effects of Displacement.*—When a plane surface of area $A'$ square feet, abutting on soil, is displaced in a normal direction by an amount $\rho$ inches, the soil will resist the displacement with a force $F$ which, on the assumptions made in Section 1 is determined by the formula:

$$F = \rho\,K\,A' \qquad (15)$$

in which $K$ stands for the "soil compression constant"; that is, the resisting force in kips exerted by the soil on a surface of 1 sq ft which is displaced normally by an amount of 1 in. The resistance offered by the soil to a lateral displacement of the surface must be presumed to increase, at first, in accordance with a similar formula, the soil being stressed elastically in shear. There is the differ-

ence, however, that when the shearing stress at any point reaches a certain limit a slip takes place between the surface and the soil particles, or between the soil particles themselves, causing the stress at that point to be relieved suddenly. Instead of an elastic reaction the result is what is known as friction, representing

TABLE 8.—D-CONSTANTS FOR RADIAL DEFLECTIONS

| Force | Point (See Fig. 4) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| (*a*) *D*-Constants for Radial Components $P_r$ | | | | | | | | |
| $P_{r1}$ | −0.00215 | −0.00657 | −0.00753 | −0.00540 | −0.00115 | +0.00384 | +0.00816 | +0.01065 |
| $P_{r2}$ | −0.00657 | −0.03747 | −0.05142 | −0.04068 | −0.01252 | +0.02266 | +0.05385 | +0.07200 |
| $P_{r3}$ | −0.00753 | −0.05142 | −0.09098 | −0.08340 | −0.03397 | +0.03455 | +0.09766 | +0.13497 |
| $P_{r4}$ | −0.00540 | −0.04068 | −0.08340 | −0.09611 | −0.05254 | +0.02546 | +0.10276 | +0.14980 |
| $P_{r5}$ | −0.00115 | −0.01252 | −0.03397 | −0.05254 | −0.04653 | −0.00163 | +0.05536 | +0.09291 |
| $P_{r6}$ | +0.00384 | +0.02266 | +0.03455 | +0.02546 | −0.00163 | −0.02736 | −0.03121 | −0.02635 |
| $P_{r7}$ | +0.00816 | +0.05385 | +0.09766 | +0.10276 | +0.05536 | −0.03121 | −0.11947 | −0.16714 |
| $P_{r8}$ | +0.01065 | +0.07200 | +0.13497 | +0.14980 | +0.09291 | −0.02635 | −0.16714 | −0.26686 |
| (*b*) *D*-Constants for Tangential Components $P_t$ | | | | | | | | |
| $P_{t1}$ | −0.00016 | −0.00045 | −0.00051 | −0.00036 | −0.00007 | +0.00026 | +0.00055 | +0.00072 |
| $P_{t2}$ | −0.00199 | −0.00898 | −0.01135 | −0.00862 | −0.00234 | +0.00531 | +0.01201 | +0.01590 |
| $P_{t3}$ | −0.00487 | −0.02731 | −0.04034 | −0.03332 | −0.01128 | +0.01709 | +0.04254 | +0.05742 |
| $P_{t4}$ | −0.00749 | −0.04611 | −0.07626 | −0.07018 | −0.02868 | +0.02962 | +0.08354 | +0.11549 |
| $P_{t5}$ | −0.00883 | −0.05696 | −0.10033 | −0.10106 | −0.04934 | +0.03466 | +0.11625 | +0.16554 |
| $P_{t6}$ | −0.00830 | −0.05502 | −0.10043 | −0.10693 | −0.05979 | +0.02846 | +0.12179 | +0.18015 |
| $P_{t7}$ | −0.00590 | −0.03970 | −0.07392 | −0.08119 | −0.04906 | +0.01634 | +0.09146 | +0.14193 |
| $P_{t8}$ | −0.00213 | −0.01445 | −0.02715 | −0.03024 | −0.01891 | +0.00505 | +0.03359 | +0.05423 |

the integrated value of the resistance offered at all the points of the surface. The shearing stress between soil and surface, therefore, cannot exceed a certain limit determined by the physical characteristics and the normal force pressing them together. The critical value at which the individual slips set in may drop almost to zero, in sandy soils, when the soil is vibrated, the hold of the particles being thereby relaxed momentarily.

*18. Tangential Forces.*—Lateral shear between a tunnel lining and the abutting soil would evidently give rise to tangential forces. However, in tunneling by the shield method, some special features must be considered in connection with shearing stresses. As the shield is being shoved ahead for the erection of another ring, an annular void is formed around the lining, and the soil, which before the shove surrounded the upper tail end of the shield, is thereby left without a support; but this does not necessarily lead to collapse, as arch action and cohesion combine to keep the bulk of the soil in place until the void has been filled with gravel which is injected from the working chamber by air pressure. As an additional precaution, grout is next forced into the gravel. The mixture of gravel and grout is in a more or less plastic state for a day or two and is thus incapable of resisting shearing forces. For some distance back of the shield the tunnel rings, therefore, are subjected to the major onset

of the soil pressure at a time when only radial pressures can be transmitted through the gravel and grout filling. Moreover, any tangential forces developed later eventually spend themselves because of their distribution around the lining, being directed clockwise as well as counterclockwise. During the slips induced by earth tremors and vibrations from the numerous small cave-ins taking place in the surrounding ground as it gradually settles, the forces will thus impel the soil particles in opposite directions. Through these tiny movements the soil particles are crowded together on some parts of the periphery, the pressure between them increasing until it balances the impelling force, at which point the movements come to a stop. The final outcome is that the shearing forces between the soil and the ring surface are replaced by tangential compressive forces in the adjacent ground, the lining at this stage performing a function similar to that of the scaffolding or "centering" used in erecting a masonry arch.

On the basis of the foregoing reasoning, one is justified in disregarding the tangential components of forces due to soil pressure, the only tangential components to be considered being those due to the weight of the lining.

*19. Soil Reactions Due to Settlement and Deflection.*—Loading a lining embedded in homogeneous soil has the effect of displacing it in the direction of the resultant of the active forces. With the assumed preponderance of downward forces, a symmetrically loaded lining is thus displaced vertically downward, the underlying soil being thereby compressed until soil reactions in sufficient amount to balance the downward forces have been developed.

The soil reactions are not determined by the settlement alone, however. Under the influence of all the forces, active as well as reactive, the lining deflects. The radial deflection at any point will either increase or reduce the compression of the soil at that point, and thus add to or detract from the soil reaction due to settlement.

In Fig. 9, let the solid circle c represent the position of a ring before it is loaded, and the line d its position and shape upon completion of the downward movement. Wherever line d is on the outside of circle c the soil is being compressed; at all other points voids are created which are presently filled as the soil closes in upon the ring under the push of active forces.

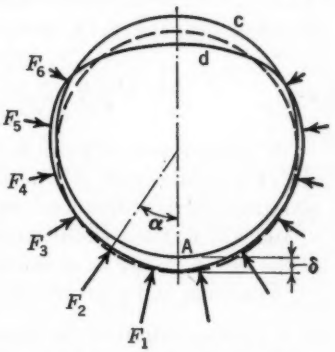


FIG. 9.—DISTRIBUTION OF SOIL REACTIONS

Settlement is defined in this paper as the distance $\delta$ by which the invert joint A has been displaced. In the imaginary case of a perfectly rigid ring there could be no deflection; the soil reactions would here be due to settlement only and would be confined to the lower half of the ring (see dashed circle in Fig. 9), the middle point of each of the lower segments being displaced radially by an amount $\delta \cos \alpha$. As the flexibility increases, however, deflection becomes more and more of a contributing factor,

and the range within which the soil is being compressed gradually extends over the upper half of the ring, giving rise to soil reactions of a downward trend, such as $F_5$ and $F_6$ in Fig. 9. Far from resisting the active force resultant, such forces add to its intensity, thereby increasing the settlement.

Referring to the left-hand half of a symmetrically loaded ring, let $u$ be the number of segments in which soil reactions are developed (that is, those whose middle points are outside of the circle c in Fig. 9). In most practical cases these segments will be consecutive from the invert point up. For a very stiff ring $u = 4$, whereas for a perfectly flexible one it would be 8. For the rings used in tunnels it is usually 5 or 6; the value is not known definitely beforehand but is tentatively assumed, the result of the computations being relied on to show whether the choice made was a correct one.

Like the active pressures (see Section 7), the soil reactions are replaced by forces $F$ applied at the middle points of the $u$ segments, the forces being considered proportional to the radial displacement at those points. Since the displacement at point $n$ is equal to the amount due to settlement, $\delta \cos \alpha_n$, plus the radial deflection $d_n$, Eq. 15 yields, for the soil reaction developed in any of the $u$ segments:

$$F_n = (\delta \cos \alpha_n + d_n)\, K\, A' \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which $A'$ is the area of the segment as projected on its chordal plane, the width of the segment being, as before, 1 ft.

*20. Evaluation of Soil Reactions.*—Solving for $d_n$ in Eq. 16 gives, for each of the $u$ segments:

$$d_n = \frac{F_n}{K\, A'} - \delta \cos \alpha_n \ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

Taking into account both active forces and soil reactions, the same deflection $d_n$, according to Eq. 14, may be expressed as:

$$d_n = \frac{r^3}{E\, I}\left[\sum_1^8 (P_r\, D_n) + \sum_1^8 (P_t\, D_n) + \sum_1^u (F\, D_n)\right] \ldots\ldots\ldots (17b)$$

If the amounts of the $u$ forces $F$ were known, their substitution in Eqs. 17*a* and 17*b* would yield identical values for the deflection $d_n$. Conversely, by equating the right-hand side of the two equations for each one of the deflections $d_n$ and solving the $u$ simultaneous equations thus obtained, the particular values of the forces $F$ may be determined by which such equality is brought about.[5]

Assuming soil reactions to be developed in segments 1 to 5 only (that is, $u = 5$), the equation for point 2, say, is: $\frac{F_2}{K\, A'} - \delta \cos \alpha_2 = \frac{r^3}{E\, I}[\sum (P_r\, D_2) + \sum (P_t\, D_2) - 0.00657\, F_1 - 0.03747\, F_2 \cdots - 0.01252\, F_5]$; or, by rearranging the terms:

$$-0.00657\, F_1 - \left(0.03747 + \frac{E\, I}{r^3\, K\, A'}\right) F_2 \cdots - 0.01252\, F_5$$
$$= -\frac{E\, I}{r^3} \cos \alpha_2\, \delta - \sum (P_r\, D_2) - \sum (P_t\, D_2) \ldots\ldots\ldots\ldots (18)$$

[5] Compare: "Soil Pressure Distribution Along Flexible Foundations," by Anders Bull, *Journal of the Franklin Institute*, June, 1942, p. 562.

Denoting by $Q$ the quantity $\frac{E\,I}{r^3\,K\,A'}$, and by $-\sum(P\,D_n)$ the terms $-\sum(P_r\,D_n)-\sum(P_t\,D_n)$, for brevity, one may write for the complete set of five simultaneous equations:

$$\left.\begin{aligned}
-(0.00215+Q)\,F_1-0.00657\,F_2-0.00753\,F_3\\
-0.00540\,F_4-0.00115\,F_5=-\frac{E\,I}{r^3}\cos\alpha_1\,\delta-\sum(P\,D_1)\ldots\ldots\\
-0.00657\,F_1-(0.03747+Q)\,F_2-0.05142\,F_3\\
-0.04068\,F_4-0.01252\,F_5=-\frac{E\,I}{r^3}\cos\alpha_2\,\delta-\sum(P\,D_2)\ldots\ldots\\
-0.00753\,F_1-0.05142\,F_2-(0.09098+Q)\,F_3\\
-0.08340\,F_4-0.03397\,F_5=-\frac{E\,I}{r^3}\cos\alpha_3\,\delta-\sum(P\,D_3)\ldots\ldots\\
-0.00540\,F_1-0.04068\,F_2-0.08340\,F_3\\
-(0.09611+Q)\,F_4-0.05254\,F_5=-\frac{E\,I}{r^3}\cos\alpha_4\,\delta-\sum(P\,D_4)\ldots\\
-0.00115\,F_1-0.01252\,F_2-0.03397\,F_3-0.05254\,F_4\\
-(0.04653+Q)\,F_5=-\frac{E\,I}{r^3}\cos\alpha_5\,\delta-\sum(P\,D_5)\ldots\ldots\ldots
\end{aligned}\right\}\ldots(19)$$

On the left-hand side of the equality signs the coefficients for the unknown forces $F$ are simply the $D$-constants given in the first five lines and columns of Table 8($a$), except that the quantity $-Q$ has been added to the $D$-constants forming the diagonal row (underscored in Table 8($a$)).

On the right-hand side all the quantities, except the settlement $\delta$, are fixed by the dimensions and other data given in the specific case. Hence, in solving for the forces $F$ expressions are obtained having the form:

$$F_n=C'_n\,\delta+C''_n\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

in which $C'_n$ and $C''_n$ are numerical quantities. From these expressions the unknown $\delta$ is determined by the conditions of equilibrium requiring that the sum of the vertical components of the soil reactions, $\sum_1^u(F\cos\alpha)$, must balance the sum of the vertical components of the active forces, or: $\sum_1^u(C'\,\delta\cos\alpha)+\sum_1^u(C''\cos\alpha)=-\sum_1^s(P_r\cos\alpha)-\sum_1^s(P_t\sin\alpha)$; from which:

$$\delta=\frac{-\sum_1^u(C''\cos\alpha)-\sum_1^s(P_r\cos\alpha)-\sum_1^s(P_t\sin\alpha)}{\sum_1^u(C'\cos\alpha)}\ldots\ldots\ldots(21)$$

Substituting this value for $\delta$ in Eq. 20, the forces $F$ are determined. The active forces $P$ being known, the moments and thrusts in the lining are found by means of Table 5, and from the values thus obtained the stresses are computed.

## Practical Application

*21. Specified Conditions.*—The proposed method will now be applied to an actual case. A tunnel lining will be designed for the identical conditions prevailing for a short stretch of tunnel, forming part of the Sixth Avenue line of the Independent Subway System of New York, N. Y. As the latter was designed by the customary method the results obtained by the two methods may be compared.

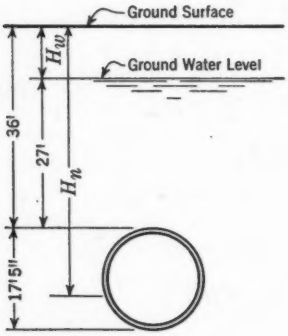


Fig. 10.—Cross Section of Tunnel

The crown of the lining is 36 ft below the ground surface and 27 ft below the ground-water level, as shown in Fig. 10. The governing data are:

| | |
|---|---|
| Radius $r_o$ of the outer surface, in feet. | 8.71 |
| Radius $r$ of the neutral circle, in inches | 102.1 |
| Assumed Weight $W$, in Kips per Cubic Foot of— | |
| Soil above ground-water level | 0.100 |
| Soil below ground-water level | 0.125 |
| Water | 0.063 |

Denoting by $H_w$ and $H_n$, respectively, the depth below ground surface of the water table and any of the points 1 to 16 in Fig. 4, the vertical and horizontal pressures $p$, in kips per square foot at points 1 to 8 (or 16 to 9), are determined by the following formulas, pressures directed upward being considered positive:

Vertical pressure at points 1 to 4—

$$p_V = + 0.063\,(H_n - H_w) \quad (22a)$$

vertical pressure at points 5 to 8—

$$p_V = - 0.100\,H_w - 0.125\,(H_n - H_w) \quad (22b)$$

and horizontal pressure at points 1 to 8—

$$p_H = + \frac{0.100\,H_n}{3} + 0.063\,(H_n - H_w) \quad (22c)$$

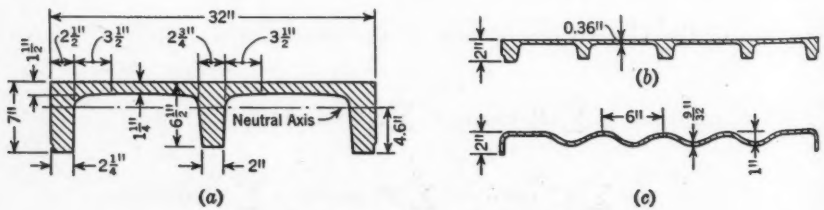


Fig. 11.—Axial Cross Section of Tunnel Segments

The lining is of cast iron for which a modulus $E$ of 12,000 kips per sq in. is assumed. It has an inner covering of concrete which, however, is disregarded

as to strength. Fig. 11(*a*) shows an axial cross section of the segments used whose physical properties per foot of tunnel are:

| | |
|---|---|
| Area $A$, deducting area of holes (square inches) | 27.6 |
| Gross moment of inertia $I$ (in.$^4$) | 136 |
| Net Section Modulus $S$ (in.$^3$)— | |
| Intrados | 23.4 |
| Extrados | 55.0 |
| Weight $W$, including concrete (kips per square foot of lining surface) | 0.22 |
| Soil compression constant $K$ (assumed) | 12 |

*22. Active Forces.*—By means of Eqs. 22, the vertical and horizontal pressures at points 1 to 8 of the lining (Fig. 4) are determined as in Cols. 3 and 4, Table 9. The pressures are multiplied by the projected area of each segment

TABLE 9.—COMPUTATIONS FOR DETERMINING RADIAL AND TANGENTIAL FORCES

| Point | Depth $H_n$, in ft | Pressure, in Kips per Sq Ft | | Projected Area, in Sq Ft | | Forces, in Kips | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Vertical | Horizontal | Horizontal | Vertical | Vertical | Horizontal | $P_r$ | $P_t$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 1 | 53.25 | +2.788 | 4.563 | 3.333 | 0.663 | +8.541 | 3.025 | 8.966 | −0.147 |
| 2 | 51.95 | +2.706 | 4.438 | 2.826 | 1.888 | +6.894 | 8.379 | 10.388 | −0.418 |
| 3 | 49.55 | +2.555 | 4.206 | 1.888 | 2.826 | +4.071 | 11.886 | 12.144 | −0.625 |
| 4 | 46.41 | +2.357 | 3.904 | 0.663 | 3.333 | +0.811 | 13.012 | 12.920 | −0.738 |
| 5 | 43.01 | −5.151 | 3.576 | 0.663 | 3.333 | −4.167 | 11.921 | 12.505 | −0.738 |
| 6 | 39.87 | −4.759 | 3.274 | 1.888 | 2.826 | −9.737 | 9.251 | 13.102 | −0.625 |
| 7 | 37.47 | −4.459 | 3.040 | 2.826 | 1.888 | −13.351 | 5.741 | 14.290 | −0.418 |
| 8 | 36.17 | −4.296 | 2.917 | 3.333 | 0.663 | −15.071 | 1.934 | 15.159 | −0.147 |

(1 ft wide) in a horizontal and vertical plane, respectively, and the amount $-\frac{2\pi \times 8.71 \times 0.22}{16} = -0.752$ kip is added to each vertical force for the weight of the lining, yielding the values given in Cols. 7 and 8, Table 9. The forces $P_r$ (Col. 9) are the sums of the radial components of each force pair, $P$ (vertical) $\cos\alpha + P$ (horizontal) $\sin\alpha$, whereas the forces $P_t$, as stated in Section 18, are the tangential components of the weight of the lining only, $-0.752 \sin\alpha$. Points 1 to 8 being located as in Fig. 4, the respective values of $\cos\alpha$ and $\sin\alpha$ are:

$$\cos\alpha_1 = -\cos\alpha_8 = \sin\alpha_4 = \sin\alpha_5 = 0.98079$$
$$\cos\alpha_2 = -\cos\alpha_7 = \sin\alpha_3 = \sin\alpha_6 = 0.83147$$
$$\cos\alpha_3 = -\cos\alpha_6 = \sin\alpha_2 = \sin\alpha_7 = 0.55557$$
$$\cos\alpha_4 = -\cos\alpha_5 = \sin\alpha_1 = \sin\alpha_8 = 0.19509$$

*23. Simultaneous Equations.*—The values found are substituted for $P_{r1}$ to $P_{r8}$ in the first column of Table 8, and each force is multiplied by the eight $D$-constants in the same line, the products being entered, in Table 10, in the

same order as the $D$-constants. The products are next added up vertically, the sums representing the terms $\sum(P\,D_n)$ on the right-hand side of Eqs. 19.

The coefficients for $\delta$ are found by multiplying the quantity $\frac{E\,I}{r^3} = \frac{12{,}000 \times 136}{1{,}064{,}331} = 1.533$ by the successive values, $\cos\alpha_1$, $\cos\alpha_2$, $\cdots$.

TABLE 10.—DEFLECTION PRODUCTS DUE TO ACTIVE FORCES

| Force | Point | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Radial: | | | | | | | | |
| 8.966 | −0.0193 | −0.0589 | −0.0675 | −0.0484 | −0.0103 | +0.0344 | +0.0732 | +0.0955 |
| 10.388 | −0.0682 | −0.3892 | −0.5342 | −0.4226 | −0.1301 | +0.2354 | +0.5594 | +0.7479 |
| 12.144 | −0.0914 | −0.6244 | −1.1049 | −1.0128 | −0.4125 | +0.4196 | +1.1860 | +1.6391 |
| 12.920 | −0.0698 | −0.5256 | −1.0775 | −1.2417 | −0.6788 | +0.3289 | +1.3277 | +1.9354 |
| 12.505 | −0.0144 | −0.1566 | −0.4248 | −0.6570 | −0.5819 | −0.0204 | +0.6923 | +1.1618 |
| 13.102 | +0.0503 | +0.2969 | +0.4527 | +0.3336 | −0.0214 | −0.3585 | −0.4089 | −0.3452 |
| 14.290 | +0.1166 | +0.7695 | +1.3956 | +1.4684 | +0.7911 | −0.4460 | −1.7072 | −2.3884 |
| 15.159 | +0.1614 | +1.0914 | +2.0460 | +2.2708 | +1.4084 | −0.3994 | −2.5337 | −4.0453 |
| Tangential: | | | | | | | | |
| −0.147 | 0.0000 | +0.0001 | +0.0001 | +0.0001 | 0.0000 | 0.0000 | −0.0001 | −0.0001 |
| −0.418 | +0.0008 | +0.0038 | +0.0047 | +0.0036 | +0.0010 | −0.0022 | −0.0050 | −0.0066 |
| −0.625 | +0.0030 | +0.0171 | +0.0252 | +0.0208 | +0.0071 | −0.0107 | −0.0266 | −0.0359 |
| −0.738 | +0.0055 | +0.0340 | +0.0563 | +0.0518 | +0.0212 | −0.0219 | −0.0617 | −0.0852 |
| −0.738 | +0.0065 | +0.0420 | +0.0740 | +0.0746 | +0.0364 | −0.0256 | −0.0858 | −0.1222 |
| −0.625 | +0.0052 | +0.0344 | +0.0628 | +0.0668 | +0.0374 | −0.0178 | −0.0761 | −0.1126 |
| −0.418 | +0.0025 | +0.0166 | +0.0309 | +0.0339 | +0.0205 | −0.0068 | −0.0382 | −0.0593 |
| −0.147 | +0.0003 | +0.0021 | +0.0040 | +0.0044 | +0.0028 | −0.0007 | −0.0049 | −0.0080 |
| $\Sigma(P\,D)$ | +0.0890 | +0.5532 | +0.9434 | +0.9463 | +0.4909 | −0.2917 | −1.1096 | −1.6291 |

Since each of the sixteen segments is spanning an angle of 22° 30′, its chordal projection has an area of: $A' = 2\,r_o \sin\frac{22.5°}{2} = 3.40$ sq ft, from which $K\,A' = 12 \times 3.40 = 40.80$; and $Q = \frac{E\,I}{r^3\,K\,A'} = \frac{1.533}{40.80} = 0.03758$.

The lining shown in Fig. 11(*a*) is considered relatively stiff and, therefore, the low value of 5 is assumed tentatively for $u$. Consequently there are five simultaneous equations for determining the soil reactions $F$, the equations, according to the foregoing, taking the form:

$$\left.\begin{aligned}
-0.03973\,F_1 - 0.00657\,F_2 - 0.00753\,F_3 - 0.00540\,F_4 - 0.00115\,F_5 &= -1.50390\,\delta - 0.08900 \\
-0.00657\,F_1 - 0.07505\,F_2 - 0.05142\,F_3 - 0.04068\,F_4 - 0.01252\,F_5 &= -1.27500\,\delta - 0.55320 \\
-0.00753\,F_1 - 0.05142\,F_2 - 0.12856\,F_3 - 0.08340\,F_4 - 0.03397\,F_5 &= -0.85190\,\delta - 0.94340 \\
-0.00540\,F_1 - 0.04068\,F_2 - 0.08340\,F_3 - 0.13369\,F_4 - 0.05254\,F_5 &= -0.29920\,\delta - 0.94630 \\
-0.00115\,F_1 - 0.01252\,F_2 - 0.03397\,F_3 - 0.05254\,F_4 - 0.08411\,F_5 &= +0.29920\,\delta - 0.49090
\end{aligned}\right\} \quad (23)$$

*24. Soil Reactions.*—The $F$-values satisfying Eqs. 23 are listed in Cols. 4 and 5, Table 11, which also shows the steps by which they are reduced to numerical terms. Proceeding in accordance with Eq. 21 and selecting proper values from

Table 11, $\delta$ is computed as: $\delta = \frac{-5.156 + 9.769 + 3.010}{+48.857} = +0.156$ in.; and finally, by substitution for $\delta$ in Col. 6, the soil reactions are determined as given in Col. 10, Table 11. The sum of their vertical components $F \cos \alpha$ (Col. 11, Table 11) is + 12.778, which tallies with $-\sum(P_r \cos \alpha) - \sum(P_t \sin \alpha) = +9.769 + 3.010$, as required for equilibrium (see derivation of Eq. 21).

TABLE 11.—COMPUTATIONS FOR SOIL REACTIONS

| Point | $P_r \cos \alpha$ ($P_r$ from Table 9) | $P_t \sin \alpha$ ($P_t$ from Table 9) | $F$ (Eq. 20) = $C'\delta + C''$ | | $F \cos \alpha = C'\delta \cos \alpha + C'' \cos \alpha$ | | $F =$ $C'\delta + C'' = F$ | | | $F \cos \alpha$ |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| 1 | +8.794 | −0.029 | +35.565 $\delta$ | +0.606 | +34.882 $\delta$ | +0.594 | +5.548 | +0.606 | 6.154 | +6.036 |
| 2 | +8.638 | −0.232 | +15.099 $\delta$ | +2.799 | +12.555 $\delta$ | +2.327 | +2.355 | +2.799 | 5.154 | +4.285 |
| 3 | +6.747 | −0.520 | +1.702 $\delta$ | +3.597 | +0.946 $\delta$ | +1.998 | +0.266 | +3.597 | 3.863 | +2.146 |
| 4 | +2.521 | −0.724 | −2.800 $\delta$ | +3.184 | −0.546 $\delta$ | +0.621 | −0.437 | +3.184 | 2.747 | +0.536 |
| 5 | −2.440 | −0.724 | −5.230 $\delta$ | +1.970 | +1.020 $\delta$ | −0.384 | −0.816 | +1.970 | 1.154 | −0.225 |
| 6 | −7.279 | −0.520 | .... | .... | .... | .... | .... | .... | .... | .... |
| 7 | −11.882 | −0.232 | .... | .... | .... | .... | .... | .... | .... | .... |
| 8 | −14.868 | −0.029 | .... | .... | .... | .... | .... | .... | .... | .... |
| $\Sigma$ | −9.769 | −3.010 | .... | .... | +48.857 $\delta$ | +5.156 | .... | .... | .... | +12.778 |

It remains to be ascertained whether the tentative assumption, $u = 5$, is correct. For that purpose the $D$-products of the soil reactions are determined in Table 12, the $F$-values being substituted for the forces $P$ in Table 8. To the

TABLE 12.—CHECK COMPUTATIONS FOR THE CONSTANT, $u$

| Line No. | Soil reaction | Point | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 6.154 | −0.0132 | −0.0404 | −0.0463 | −0.0332 | −0.0071 | +0.0236 | +0.0502 | +0.0655 |
| 2 | 5.154 | −0.0339 | −0.1931 | −0.2650 | −0.2097 | −0.0645 | +0.1168 | +0.2775 | +0.3711 |
| 3 | 3.863 | −0.0291 | −0.1986 | −0.3515 | −0.3222 | −0.1312 | +0.1335 | +0.3773 | +0.5214 |
| 4 | 2.747 | −0.0148 | −0.1117 | −0.2291 | −0.2640 | −0.1443 | +0.0699 | +0.2823 | +0.4115 |
| 5 | 1.154 | −0.0013 | −0.0144 | −0.0392 | −0.0606 | −0.0537 | −0.0019 | +0.0639 | +0.1072 |
| 6 | Total, lines 1–5 | −0.0923 | −0.5582 | −0.9311 | −0.8897 | −0.4008 | +0.3419 | +1.0512 | +1.4767 |
| 7 | $\Sigma(P\ D)$ | +0.0890 | +0.5532 | +0.9434 | +0.9463 | +0.4909 | −0.2917 | −1.1096 | −1.6291 |
| 8 | Line 6 + line 7 | −0.0033 | −0.0050 | +0.0123 | +0.0566 | +0.0901 | +0.0502 | −0.0584 | −0.1524 |
| 9 | Line 8 × 0.6522 | −0.0022 | −0.0033 | +0.0080 | +0.0369 | +0.0588 | +0.0327 | −0.0381 | −0.0994 |
| 10 | $\delta \cos \alpha$ | +0.1530 | +0.1297 | +0.0867 | +0.0304 | −0.0304 | −0.0867 | −0.1297 | −0.1530 |
| 11 | Line 9 + line 10 | +0.1508 | +0.1264 | +0.0947 | +0.0673 | +0.0284 | −0.0540 | −0.1678 | −0.2524 |
| 12 | Line 11 × 40.8 | 6.153 | 5.157 | 3.863 | 2.746 | 1.159 | .... | .... | .... |

sums of these products in line 6, Table 12, are added the amounts obtained for $\sum(P\ D)$ in Table 10, and the totals (line 8) are multiplied by the factor $\frac{r^3}{E\ I} = \frac{1{,}064{,}331}{12{,}000 \times 136} = 0.6522$, yielding the deflections, in inches, at points 1 to 8 (line 9, Table 12). Adding to each deflection the radial displacement due to settlement, $\delta \cos \alpha_n$, the amounts are found (line 10) by which the points have moved out from their neutral position on circle c in Fig. 9. Only at points

where this amount is positive are soil reactions developed. Table 12 shows that the displacement (line 11) is positive at points 1 to 5 only, proving that the tentative assumption, $u = 5$, is correct. This table provides a rigorous check on the correctness of the arithmetical work since, according to Eq. 16, the radial displacement at any of the points 1 to 5, multiplied by the factor $K A' = 40.8$, should give the soil reaction at that point. As shown in line 12, Table 12, this is the case within the limits of unavoidable error.

*25. Bending Moments and Thrusts.*—The bending moments and thrusts produced in the ring may now be determined, as shown in Table 13, active forces

TABLE 13.—COMPUTATION OF BENDING MOMENTS AND THRUSTS IN THE TUNNEL RING

| Force | Moments at Joints | | | | | | | | | Thrusts | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | J | Joint F | Joint J |
| (*a*) Active Forces | | | | | | | | | | | |
| Radial: | | | | | | | | | | | |
| 8.966 | +1.437 | +5.867 | +9.930 | +11.378 | +10.336 | +7.288 | +3.017 | −1.553 | −5.544 | +11.213 | +0.109 |
| 10.388 | +4.132 | +6.371 | +8.475 | +10.299 | +9.649 | +7.027 | +3.149 | −1.128 | −4.974 | +11.386 | +1.082 |
| 12.144 | +5.224 | +6.791 | +6.040 | +5.778 | +6.254 | +5.091 | +2.810 | −0.030 | −2.857 | −9.778 | +3.155 |
| 12.920 | +3.818 | +4.058 | +2.832 | +0.271 | −0.438 | +0.904 | +1.468 | +1.353 | +0.629 | +5.323 | +5.544 |
| 12.505 | +0.609 | −0.558 | −2.030 | −3.633 | −5.303 | −4.245 | −0.709 | +2.061 | +3.695 | −0.474 | +6.899 |
| 13.102 | −3.083 | −5.604 | −7.262 | −7.958 | −7.810 | −7.217 | −3.777 | +1.756 | +5.636 | −3.824 | +7.489 |
| 14.290 | −6.842 | −10.646 | −12.472 | −12.287 | −10.489 | −7.788 | −5.144 | −0.327 | +5.685 | −4.686 | +6.451 |
| 15.159 | −9.373 | −14.005 | −15.926 | −15.150 | −12.259 | −8.236 | −4.238 | −1.460 | +2.430 | −5.222 | +2.773 |
| Tangential: | | | | | | | | | | | |
| −0.147 | −0.003 | −0.020 | −0.033 | −0.037 | −0.034 | −0.024 | −0.010 | +0.005 | +0.018 | −0.036 | 0.000 |
| −0.418 | −0.057 | −0.156 | −0.255 | −0.295 | −0.270 | −0.192 | −0.081 | +0.039 | +0.144 | −0.298 | −0.009 |
| −0.625 | −0.191 | −0.383 | −0.537 | −0.626 | −0.584 | −0.423 | −0.187 | +0.072 | +0.304 | −0.682 | −0.057 |
| −0.738 | −0.334 | −0.583 | −0.742 | −0.802 | −0.763 | −0.573 | −0.272 | +0.069 | +0.386 | −0.984 | −0.167 |
| −0.738 | −0.386 | −0.623 | −0.751 | −0.765 | −0.685 | −0.535 | −0.282 | +0.029 | +0.334 | −1.039 | −0.311 |
| −0.625 | −0.304 | −0.470 | −0.548 | −0.537 | −0.455 | −0.334 | −0.198 | −0.015 | +0.191 | −0.202 | −0.404 |
| −0.418 | −0.144 | −0.216 | −0.248 | −0.237 | −0.194 | −0.134 | −0.073 | −0.021 | +0.057 | −0.084 | −0.356 |
| −0.147 | −0.018 | −0.027 | −0.031 | −0.029 | −0.024 | −0.016 | −0.008 | −0.002 | +0.003 | −0.010 | −0.144 |
| $\Sigma(P\,B)$ | −5.515 | −10.204 | −13.558 | −14.630 | −13.069 | −9.407 | −4.535 | +0.848 | +6.137 | +20.159 | +32.054 |
| (*b*) Soil Reactions and Totals | | | | | | | | | | | |
| 6.154 | +0.987 | +4.027 | +6.816 | +7.809 | +7.095 | +5.002 | +2.071 | −1.066 | −3.805 | +7.696 | +0.075 |
| 5.154 | +2.050 | +3.161 | +4.205 | +5.110 | +4.788 | +3.487 | +1.562 | −0.560 | −2.468 | +5.649 | +0.537 |
| 3.863 | +1.662 | +2.160 | +1.921 | +1.838 | +1.989 | +1.619 | +0.894 | −0.010 | −0.909 | +3.110 | +1.004 |
| 2.747 | +0.812 | +0.863 | +0.602 | +0.058 | −0.093 | +0.192 | +0.312 | +0.288 | +0.134 | +1.132 | +1.179 |
| 1.154 | +0.056 | −0.051 | −0.187 | −0.335 | −0.489 | −0.392 | −0.065 | +0.190 | +0.341 | −0.042 | +0.637 |
| $\Sigma(F\,B)$ | +5.567 | +10.160 | +13.357 | +14.480 | +13.290 | +9.908 | +4.774 | −1.158 | −6.707 | +17.545 | +3.432 |
| $\Sigma(P\,B)$ | −5.515 | −10.204 | −13.558 | −14.630 | −13.069 | −9.407 | −4.535 | +0.848 | +6.137 | +20.159 | +32.054 |
| Total | +0.052 | −0.044 | −0.201 | −0.150 | +0.221 | +0.501 | +0.239 | −0.310 | −0.570 | +37.704 | +35.486 |
| × 102.1 | +5.3 | −4.5 | −20.5 | −15.3 | +22.6 | +51.2 | +24.4 | −31.7 | −58.2 | .... | .... |

and soil reactions having been treated separately. The procedure is the same as in finding the deflections: Each force is multiplied by the nine $B_M$-constants appearing in the same line in Table 5(*a*), and the products, entered in the order of the constants, are added. The product sums $\sum(P\,B)$ due to the active forces are then added to those due to soil reactions $\sum(F\,B)$ (Table 13(*b*)), and the sums are multiplied by the radius $r = 102.1$ in. Thus, for the moment at the various joints, the amounts given in the last line of Table 13(*b*) are obtained.

The thrusts are determined only for joints F and J, where the stresses reach critical values. The last two columns of Table 13 are used for that purpose, the products being those of the forces in the $B_T$-constants of Tables 5(*c*) and 5(*d*).

Table 13 illustrates what was stated in Section 2 in regard to the sensitive state of moment balance in a ring. At joint B (Fig. 4), for example, the $B_M$-products due to forces $P_{r3}$ and $P_{r8}$ are 6.791 and − 14.005, respectively, as compared to the total of 0.044.

*26. Critical Stresses.*—The design of a lining is governed by the maximum stresses caused by bending and thrust combined, the negative thrust due to the increased air pressure in the working chamber during construction being also considered. With a cross section like the one in Fig. 11(*a*), the greater part of the maximum stresses are due to bending, the fibers most highly stressed being those in the ribs at the intrados.

According to Table 13(*b*), the moment in the present case reaches a positive maximum at joint F, and a negative maximum at joint J. For the properties given in Section 21, therefore, the respective bending stresses $\sigma_b$, expressed in kips per square inch, are: $\sigma_{bF} = \frac{51.2}{23.4} = -2.19$; and $\sigma_{bJ} = \frac{58.2}{23.4} = +2.49$—a positive sign indicating tension and a negative sign compression.

The thrusts at joints F and J are shown in Table 13(*b*) to be 37.7 kips and 35.5 kips, respectively, which yield, for the corresponding stresses $\sigma_t$: $\sigma_{tF} = \frac{37.7}{27.6} = -1.37$; and $\sigma_{tJ} = \frac{35.5}{27.6} = -1.29$.

The internal air pressure is usually made equal to the hydrostatic pressure of the ground water at the level of the tunnel axis. For a pressure head of 35.7 ft and an effective radius of 8.5 ft, the tensile force produced in the lining is $35.7 \times 0.063 \times 8.5 = 19.1$ kips, resulting in a stress of $\sigma_a = \frac{19.1}{27.6} = +0.69$.

Accordingly, the maximum tensile stress, occurring at joint J, is $\sigma_J = +2.49 - 1.29 + 0.69 = +1.89$ kips per sq in. The compressive stress at joint F does not reach its maximum until the internal air pressure is removed, when $\sigma_F = -2.19 - 1.37 = -3.56$ kips per sq in.

As the usual working stresses for cast iron are 4.0 kips per sq in. in tension and 20.0 kips per sq in. in compression, the stress at joint J is only 47% and that at joint F only 18% of the respective allowable values.

The obvious conclusion is that the lining, as designed, is too heavy; but to what extent its cross section should be reduced in order to meet the requirements of economy is not immediately apparent. A reduction in area also involves the moment of inertia, and as the latter enters into Eqs. 19 by which the soil reactions are determined, the pressure distribution will not remain the same as before. This affects the moments in particular because of their extreme sensitiveness to changes in the pressure distribution.

*27. Economic Cross Section.*—For investigating the question raised in the preceding section assume the moment of inertia of the lining, per foot of tunnel, to be reduced in a geometric progression to 1/4, 1/16, 1/64, etc., of its original value of 136 in.$^4$, the reduction being effected by shrinking the radial dimensions

of the cross section, Fig. 11(*a*). Such shrinking causes the section modulus and area to be reduced correspondingly to $(1/4)^{2/3}$, $(1/16)^{2/3}$, $(1/64)^{2/3}$ ··· and $(1/4)^{1/3}$, $(1/16)^{1/3}$, $(1/64)^{1/3}$ ··· of the original amounts of 23.4 in.³ and 27.6 in.², respectively. The soil reactions, moments, thrusts, and critical stresses are then determined for each of the reduced sections by means of the method explained.

TABLE 14.—EFFECTS OF VARIATIONS IN THE MOMENT OF INERTIA OF THE TUNNEL LINING

| Description | Moment of Inertia $I$, in Inches⁴: 136 | | 34 | | 8.5 | | 2.13 | | 0.53 | | 0.13 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *F* | *M* | *F* | *M* | *F* | *M* | *F* | *M* | *F* | *M* | *F* | *M* |
| | | | | | (*a*) Physical Properties | | | | | | | |
| $S$ (in.³) | 23.4 | | 9.29 | | 3.69 | | 1.46 | | 0.580 | | 0.227 | |
| $A$ (in.²) | 27.6 | | 17.4 | | 11.0 | | 6.90 | | 4.34 | | 2.72 | |
| | | | | | (*b*) Soil Reactions, in Kips, and Moments, in Inch-Kips | | | | | | | |
| Joint A | | +5.3 | | +4.3 | | +0.9 | | +0.2 | | 0 | | 0 |
| Point 1 | 6.154 | | 6.632 | | 7.068 | | 7.232 | | 7.286 | | 7.295 | |
| Joint B | | −4.5 | | +0.9 | | +0.6 | | −0.1 | | −0.1 | | −0.2 |
| Point 2 | 5.154 | | 5.199 | | 5.546 | | 5.674 | | 5.715 | | 5.735 | |
| Joint C | | −20.5 | | −6.6 | | −1.7 | | −0.9 | | −0.4 | | −0.5 |
| Point 3 | 3.863 | | 3.481 | | 3.639 | | 3.793 | | 3.841 | | 3.840 | |
| Joint D | | −15.3 | | −10.1 | | −2.7 | | −0.5 | | +0.2 | | +0.1 |
| Point 4 | 2.747 | | 2.713 | | 2.633 | | 2.643 | | 2.674 | | 2.698 | |
| Joint E | | +22.7 | | +1.0 | | −1.6 | | −0.8 | | 0 | | −0.1 |
| Point 5 | 1.154 | | 2.588 | | 2.982 | | 2.957 | | 2.908 | | 2.884 | |
| Joint F | | +51.2 | | +27.7 | | +8.0 | | +1.2 | | −0.5 | | −0.9 |
| Point 6 | .... | | .... | | 1.282 | | 1.941 | | 2.171 | | 2.235 | |
| Joint G | | +24.4 | | +24.5 | | +12.6 | | +7.0 | | +5.2 | | +4.6 |
| Joint H | | −31.7 | | −16.7 | | −4.9 | | −0.6 | | +0.5 | | +0.9 |
| Joint J | | −58.2 | | −38.3 | | −17.8 | | −10.9 | | −8.7 | | −8.0 |
| | | | | | (*c*) Settlement $\delta$, in Inches | | | | | | | |
| ..... | 0.156 | .... | 0.175 | .... | 0.206 | .... | 0.292 | .... | 0.624 | .... | 1.991 | .... |
| | | | | | (*d*) Thrust, in Kips | | | | | | | |
| Joint F | | 37.7 | | 37.8 | | .... | | .... | | .... | | .... |
| Joint G | | .... | | .... | | 38.0 | | 38.3 | | 38.4 | | 38.5 |
| Joint J | | 35.5 | | 36.2 | | 37.2 | | 37.6 | | 37.7 | | 37.8 |
| | | | | | (*e*) Maximum Stress at Joint F or G, in Kips per Square Inch | | | | | | | |
| $\sigma_b$ | 51.2/23.4 = −2.19 | | 27.7/9.29 = −2.98 | | 12.6/3.69 = −3.42 | | 7.0/1.46 = −4.79 | | 5.2/0.580 = −8.97 | | 4.6/0.227 = −20.26 | |
| $\sigma_t$ | 37.7/27.6 = −1.37 | | 37.8/17.4 = −2.17 | | 38.0/11.0 = −3.45 | | 38.3/6.90 = −5.55 | | 38.4/4.34 = −8.85 | | 38.5/2.72 = −14.15 | |
| Total | | −3.56 | | −5.15 | | −6.87 | | −10.34 | | −17.82 | | −34.41 |
| | | | | | (*f*) Maximum Stress at Joint J, in Kips per Square Inch | | | | | | | |
| $\sigma_b$ | 58.2/23.4 = +2.49 | | 38.3/9.29 = +4.12 | | 17.8/3.69 = +4.82 | | 10.9/1.46 = +7.47 | | 8.7/0.58 = +15.00 | | 8.0/0.227 = +35.24 | |
| $\sigma_t$ | 35.5/27.6 = −1.29 | | 36.2/17.4 = −2.08 | | 37.2/11.0 = −3.38 | | 37.6/6.90 = −5.45 | | 37.7/4.34 = − 8.70 | | 37.8/2.72 = −13.90 | |
| $\sigma_p$ | 19.1/27.6 = +0.69 | | 19.1/17.4 = +1.10 | | 19.1/11.0 = +1.74 | | 19.1/6.90 = +2.77 | | 19.1/4.34 = + 4.40 | | 19.1/2.72 = +7.02 | |
| Total | | +1.89 | | +3.14 | | +3.18 | | +4.79 | | +10.70 | | +28.36 |

Omitting the arithmetical work for the sake of brevity, the results are tabulated for ready comparison in Table 14. The substantial change in the moments attending a relatively slight variation in the soil reactions should be noted.

The effects of a successive reduction in cross section are more strikingly revealed in Fig. 12, where the moments obtained for the several cross sections have been plotted against the developed periphery of the left half of the lining,

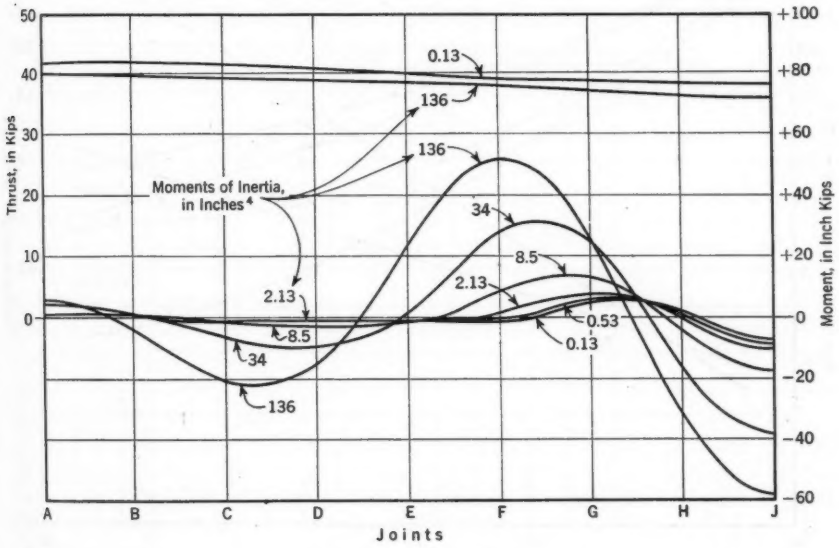


Fig. 12.—Moments and Thrusts Related to the Moment of Inertia of a Tunnel Lining

each graph being marked with the corresponding value of $I$. The thrusts, which vary but slightly, have been plotted for the largest and smallest cross section only.

Fig. 12 shows that, when the cross section is reduced beyond a certain point, the moments in the lower portion of the lining practically disappear, the only moments remaining being those produced in the arch formed by the top portion, where there are no soil reactions. In this respect the results differ sharply from those obtained by the customary method of analysis, according to which the moment at the invert joint is only slightly smaller than that at the crown.

When the reduction is accomplished, as in this case, through radial shrinking of a cross section of a given shape the relatively most economic section is evidently that at which one of the critical stresses reaches its allowable limit, although this does not necessarily mean that it is the most economic section attainable. For determining the relatively most economic section, the critical stresses, section moduli, and cross section areas in the last four columns of Table 14 have been plotted against the corresponding moments of inertia, as shown in Fig. 13, a logarithmic scale being used.

Referring to the curves drawn in solid lines, representing the case of a cast-iron lining, in order to bring the tensile stress at joint J up to the allowable 4 kips per sq in. the moment of inertia must be reduced to 3.3 in.$^4$, corresponding to an area of 8.0 sq in. This would indicate that with a lining as actually designed more than 70% of the cast iron is wasted.

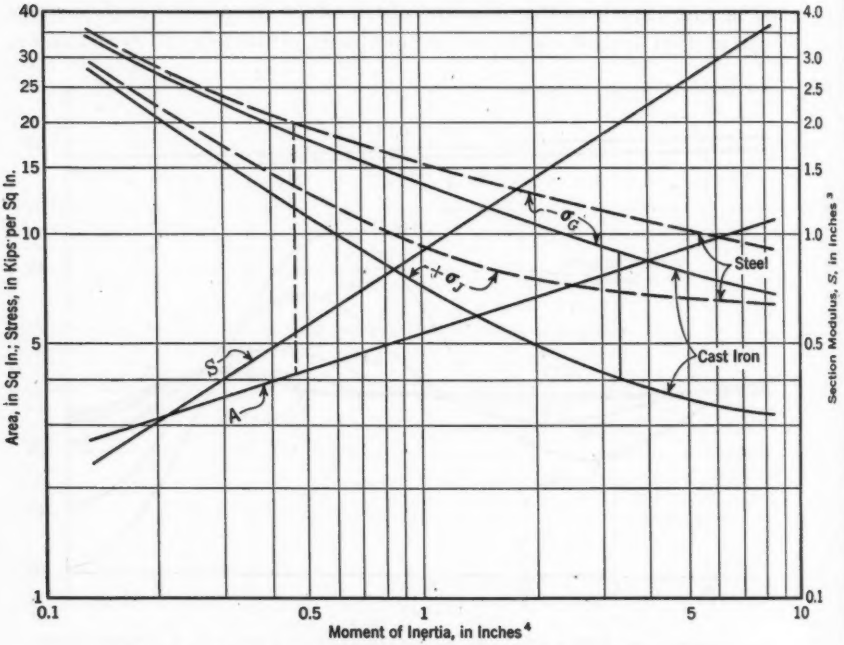


FIG. 13.—ECONOMIC SECTION DETERMINED BY A GRAPHIC INTERPOLATION

Theoretically, the cross section shown in Fig. 11(*b*) would meet requirements as regards strength. Practically, however, the casting of large segments of such a slender shape is apt to present difficulties, the cylindrical portion or "skin" being less than 3/8 in. thick.

*28. Steel Versus Cast Iron.*—By making the lining of steel, not only is the fabrication of the segments simplified, but a still further reduction of the cross section is made possible because of the greater tensile strength of that material. The consequent lighter weight of the segments will make them easier to handle. They may be made to span wider angles, so that the number of joints per ring may be reduced and erection facilitated. Because of the high ductility of steel, cracking of segments, always a source of serious trouble, is eliminated. Although cast iron is less apt to rust, a number of inexpensive treatments are now available by which steel surfaces are rendered inert toward oxygen. Steel has proved entirely satisfactory as a lining material in many instances, both in the United States and elsewhere, so one may rather wonder at the continued use of cast iron for shield-driven tunnels, when for most other structures it has long ago been replaced by steel.

On account of the higher modulus of elasticity of steel (30,000 kips per sq in., as compared to 12,000 for cast iron), a steel lining will be stiffer than a cast-iron one of the same dimensions, the effect being the same as if its moment of inertia were increased to 30,000/12,000 = 5/2 times its actual value, resulting in higher bending moments and slightly lower thrusts, as shown in Fig. 12. The stresses produced are easily determined from the data in Table 14, observing that the actual moment of inertia corresponding to the moments and thrusts in any column is thus only 2/5 = 0.4 times the value given for $I$ in that column and that, therefore, the section modulus $S$ is to be multiplied by $0.4^{2/3} = 0.542$, and the area $A$ by $0.4^{1/3} = 0.737$ when computing the stresses.

The critical stresses thus obtained for a steel lining are represented by the two curves drawn in dashed lines in Fig. 13. If working stresses of $\pm$ 20 kips per sq in. are allowed, the strength of the lining is no longer governed by the tensile stress at joint J, but by the compressive stress at joint G, the value $-$ 20 corresponding to a moment of inertia of 0.47 in.$^4$, a section modulus of 0.53 in.$^3$, and an area of 4.1 sq in. per foot of tunnel.

The cross section shown in Fig. 11(*c*) approximately meets these requirements. It has a moment of inertia of 0.58 in.$^4$, a section modulus of 0.79 in.$^3$, and an area of 3.8 sq in. per ft. The maximum stresses produced under the loading conditions assumed will be $-$ 18.0 kips per sq in. at joint G, and + 7.6 kips per sq in. at joint J. The higher economy attained as compared to the section derived from Fig. 13 is due to a more favorable distribution of the material about the neutral axis.

The foregoing results are in good agreement with the sections used in large corrugated steel culverts under comparable loading conditions, details of which have been published. A culvert of 15-ft diameter was thus installed in 1939 under an embankment of the Denver and Salt Lake Railway.[6] It has served satisfactorily under a 42-ft cover of dry fill and superimposed train load. The embankment is 20 ft wide at the top, with slopes 1 on 1.5. According to data furnished by M. G. Spangler,[7] M. Am. Soc. C. E., the corrugations have a depth of 2 in. and a spacing of 6 in., the thickness of the plates varying from 3/16 in. at the intake and outlet to 9/16 in. at the center of the embankment. The latter thickness gives a moment of inertia of 2.3 in.$^4$, a section modulus of 2.0 in.$^3$, and an area of 4.3 sq in. per ft.

Other corrugated steel culverts of 13.75-ft and 15-ft diameters with covers of 57 ft and 27 ft, respectively, were built in connection with the Lake Mead Road project.[8] For the smaller diameter the same type of plates was used as for the aforementioned railroad culvert; for the larger diameters, plates of somewhat lighter size were used.

A paper by Karl Terzaghi, M. Am. Soc. C. E., "Liner-Plate Tunnels on the Chicago Subway,"[9] may also be referred to in this connection. Discussing cases in which the soil conditions as well as the shape of the tunnel are different

[6] *Engineering News-Record*, November 7, 1940, p. 74.

[7] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Iowa State College, Ames, 1941.

[8] "Drainage Problems on New Lake Mead Road," by George D. Whittle, *Civil Engineering*, April, 1942, p. 196.

[9] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p 970.

from those considered in the present paper, Professor Terzaghi, on empirical premises, arrives at the same conclusion in regard to the excessive strength of the linings required by the customary method of design because of its failure to make proper allowance for the soil reactions.

Lieutenant-Commander Spangler[10] calls attention to what he terms "deferred deflection," representing the slow, additional deflection observed in culverts after the loading has reached its maximum. The writer considers this phenomenon as being in accord with the views expressed in Section 18 regarding the gradual transfer of the tangential soil pressures from the lining to the surrounding ground. As the pressures are directed outward, both in the upper and lower half of the lining, such transfer would naturally cause an outward yielding of the soil at either end of the horizontal diameter and a sagging at the crown, without a corresponding increase in radial loads, the flexible lining adjusting itself to these movements.

*29. Construction Features.*—In view of the light cross section required when a lining is made of steel, segments of the rolled or pressed type would be the most suitable ones. The skin and the curved ribs are here formed from a single plate, the straight, axial ribs being subsequently welded on in the shop. Bolts (which are now used for joining the segments to one another) may conceivably be dispensed with, as the small thickness of the flanges makes them particularly well adapted to resistance welding. With the welding transformer mounted at the end of the erector arm a series of "shot welds" may be produced through the flanges in a fraction of the time needed for bolting them together. By overlapping the welds, the flanges may be joined by a continuous seam of fused metal insuring a waterproof joint requiring no calking, except at corners. As no arc or flame is used with this kind of welding, no fire hazard is involved.

One objection that may be raised to reducing the weight of the lining is that a large cross section is necessary for resisting the pressure of the shoving jacks. There seems to be no good reason, however, for making material that is needed only during the erection of a tunnel an integral part of it, something that is being done in no other type of structure. A more satisfactory solution would be to use scaffolding, as it could be reclaimed after having served its purpose. The scaffolding should be tied in with the lining so as to effect a gradual transfer of the jack pressure to the surrounding ground; some distance back of the shield it would then no longer be stressed during a shove, so that it could be detached and used over again. Furthermore, it should leave the joints in the ring to be welded free and easily accessible.

Although it is true that such a type of scaffolding has yet to be developed, it may also be stated that, thus far, there has been no demand for it, as a heavy lining has been considered essential on account of the soil pressures. Once the fallacy of this contention is recognized, a solution of the problem presented by the scaffolding may be confidently expected.

## Conclusions

In designing the linings of shield-driven tunnels, the erroneous assumption is currently made that the vertical soil reactions are uniformly distributed over the

[10] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Iowa State College, Ames, 1941, p. 61.

horizontal diameter of the lining. When, instead, their distribution is determined by rational methods it is found that the stresses in the lining are much lower than heretofore presumed, permitting the cross section to be substantially reduced. Such reduction, because of the consequent increase in the flexibility of the lining, has the effect of still further reducing the stresses. By successive reductions and interpolation of the values obtained for the stresses, an economic cross section may be determined.

Steel is more economical than cast iron as a lining material because of its higher tensile strength. Steel segments are simpler to manufacture and may be joined to one another by means of resistance welding, thus facilitating erection and waterproofing of the tunnel.

In order that the prospective savings in material and labor may be realized, however, a suitable form of scaffolding must first be developed.

## APPENDIX

### Notation

The letter symbols, used in this paper, conform essentially to American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[11] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932. Symbols for soil characteristics conform to "Soil Mechanics Nomenclature."[12] Subscript $n$, standing for any number from 1 to 16, is used for identifying any one of the sixteen imaginary segments of a tunnel ring; and subscript $N$, standing for any letter from A to R, serves to identify the joint between any two such segments (see Fig. 4). Subscripts $T$, $V$, and $M$ signify "thrust," "shear," and "moment," respectively; and $r$ and $t$ signify "radial component" and "tangential component," respectively. Symbols are defined where they first appear; only those entering the formulas used in solving specific problems are listed in this Appendix.

$A_N$ = a constant used in determining reactions at the invert joint A;
$A$ = axial cross section of lining per foot of tunnel, in square inches;
$A'$ = area of a segment as projected on its chordal plane, in square feet;
$B_N$ = a constant used with symmetrical loading in determining reactions at any of the joints A to J;
$C_n$ = a constant used in determining radial deflection at any of the points 1 to 16;
$D_n$ = a constant used with symmetrical loading in determining radial deflection at any of the points 1 to 8;
$d$, $d_r$ = radial deflection, in inches; positive when directed away from center of tunnel;
$E$ = modulus of elasticity, in kips per square inch;

[11] ASA—Z10a—1932.
[12] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

$F$ = soil reaction, in kips; positive when directed toward center of tunnel;
$H_n$ = depth of any point n below ground surface, in feet;
$H_w$ = depth of water table below ground surface, in feet; considered negative when ground surface is submerged;
$I$ = average moment of inertia of lining per foot of tunnel, in inches$^4$;
$K$ = soil compression constant, in kips per square foot per inch;
$M$ = moment reaction, in inch-kips; positive when stretching outer fibers of lining;
$P$ = active load, in kips; the radial component $P_r$ is positive when directed toward center of tunnel; the tangential component $P_t$ is positive when directed upward;
$Q$ = a constant $= \dfrac{E\,I}{r^3\,K\,A'}$;
$r$ = radius of neutral circle of lining, in inches;
$r_o$ = radius of outer surface of lining, in feet;
$S$ = section modulus of lining per foot of tunnel, in inches$^3$;
$s$ = arc measured on neutral circle from point A to point $x,y$ (see Eqs. 7);
$T$ = thrust reaction, in kips; positive when due to compression;
$u$ = number of segments in which soil reactions are developed;
$V$ = shear reaction, in kips; positive when it tends to produce clockwise rotation;
$X,Y$ = rectangular coordinates of a given point; $X$ is positive to the right of the vertical diameter of the lining; $Y$ is positive above a tangent to the neutral circle in point A;
$x,y$ = rectangular coordinates of a moving point;
$\alpha, \beta$ = angles measured from point A to points p and q on neutral circle;
$\delta$ = settlement, or vertical displacement of invert point A due to loading, in inches; positive when directed downward.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

# ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES USING REDUCED EQUATIONS

By Lee H. Johnson, Jr.,[1] M. Am. Soc. C. E.

## Synopsis

The method presented in this paper is useful in simplifying the analysis of certain types of statically indeterminate structures and is based upon a simple arithmetical principle. The principle is explained first and then applied for purposes of illustration to the solution of two-hinged arches. Practical data to aid the designer of two-hinged arches are derived. Additional applications of the method are mentioned in the conclusion.

The method may be applied either to framed or to solid-type structures. It permits a close approximation of the reactions of certain statically indeterminate structures solely from the geometry of the structure—that is, without having to assume the sizes of members or sections. This method does not replace the methods in current use; nor does it apply to all structures. It does serve, in certain instances, to eliminate most of the preliminary computations and to make the solutions short and direct.

The writer emphasizes that the paper explains and illustrates the method, but does not intend to present an exhaustive study of its applications. It is presumed that the reader is familiar with the current methods for solving statically indeterminate structures.

## Notation

The letter symbols in this paper are defined where they first appear and are assembled for reference in the Appendix.

## Basic Principle

The basic principle is one of simple arithmetic which applies to a fraction of the type, $\frac{a_1 k_1 + a_2 k_2}{b_1 k_1 + b_2 k_2}$, in which corresponding terms in numerator and

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 1, 1945.

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denominator have the same multiplier; that is, $a_1$ and $b_1$ are both multiplied by $k_1$. Stated simply, the principle is that

$$\frac{a_1 k_1 + a_2 k_2}{b_1 k_1 + b_2 k_2} = \frac{a_1 + a_2}{b_1 + b_2} \quad \ldots\ldots (1)$$

provided either that $k_1 = k_2$ or that $a_1/b_1 = a_2/b_2$. This is obvious from an inspection of the difference of the two fractions, which in this paper is called the "difference term"; or, for brevity, $G$:

$$\frac{a_1 + a_2}{b_1 + b_2} - \frac{a_1 k_1 + a_2 k_2}{b_1 k_1 + b_2 k_2} = \frac{(a_1 b_2 - a_2 b_1)(k_2 - k_1)}{(b_1 + b_2)(b_1 k_1 + b_2 k_2)} = G \quad \ldots\ldots (2)$$

The two fractions on the left side are equal when $G$ is equal to zero—that is, when $a_1/b_1 = a_2/b_2$; or when $k_1 = k_2$. Extending this principle for expressions with any number of terms, $n$:

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} - \frac{a_1 k_1 + a_2 k_2 + \cdots + a_n k_n}{b_1 k_1 + b_2 k_2 + \cdots + b_n k_n}$$

$$= \frac{(a_1 b_2 - a_2 b_1)(k_2 - k_1) + \cdots + (a_{n-1} b_n - a_n b_{n-1})(k_n - k_{n-1})}{(b_1 + b_2 + \cdots b_n)(k_1 b_1 + k_2 b_2 + \cdots + k_n b_n)}$$

$$= G = 0 \quad \ldots\ldots (3)$$

provided either that $k_1 = k_2 = \cdots = k_n$, or that $a_1/b_1 = a_2/b_2 = \cdots = a_n/b_n$.

If the $k$-values are not all equal and the $(a/b)$-ratios are not all equal, the difference term will not be zero. The magnitude of $G$ depends upon the magnitude of the variations between the $k$-values and between the $(a/b)$-values. For small variations in these values and for finite values of $n$, the difference term $G$ is very small. The numerator will be small since it consists of small terms, some + and some −, formed by the products of small differences, but the denominator will be correspondingly large since it consists of the product of two sums of comparatively large terms. Expressed in terms of integrals, the principle is that—

$$\frac{\int f_1(x)\,dx}{\int f_2(x)\,dx} = \frac{\int f_1(x)\,k(x)\,dx}{\int f_2(x)\,k(x)\,dx} \quad \ldots\ldots (4)$$

—provided that $k(x) = a$, in which $a$ is an arbitrary constant; or, that $f_2(x) = b\,f_1(x)$, in which $b$ is an arbitrary constant. This is true for definite integrals provided that all integrals have the same limits of integration.

The function $k(x)$ may not be constant, although nearly so—for example, when

$$k(x) = a + \epsilon(x) \quad \ldots\ldots (5a)$$

in which $\epsilon(x)$ is a function whose absolute value can be made arbitrarily small. Furthermore, $f_1(x)$ may not have a constant coefficient as in the expression

$$f_2(x) = [b + \eta(x)]\,f_1(x) \quad \ldots\ldots (5b)$$

in which $\eta(x)$, like $\epsilon(x)$, is a function whose absolute value can be made arbi-

trarily small. Under the conditions expressed by Eqs. 5, the difference between the two fractions in Eq. 4 will not be zero, but can be expressed in the "difference term"—

$$G = \frac{\int f_1(x)\,dx \int \epsilon(x)\,\eta(x)\,f_1(x)\,dx - \int \epsilon(x)\,f_1(x)\,dx \int \eta(x)\,f_1(x)\,dx}{\int [b + \eta(x)]\,f_1(x)\,dx \int [b + \eta(x)]\,[a + \epsilon(x)]\,f_1(x)\,dx} \quad \ldots\ldots(6)$$

—simply by putting the difference of the two fractions over a common denominator.

Assume that all the integrals in the difference term are definite integrals with the same limits of integration. Also, let $f_1(x)$ be a continuous function in the region of integration, $c \leqq x \leqq d$, and let $\left| \int_c^d f_1(x)\,dx \right| \geqq B$, in which $B$ is an arbitrary constant. These conditions exist in many engineering problems involving such equations. The difference term will approach the quantity,

$$G = \frac{0}{b^2\,a \int_c^d f_1(x)\,dx \int_c^d f_1(x)\,dx} \quad \ldots\ldots(7)$$

as $|\epsilon(x)| \to 0$, and as $|\eta(x)| \to 0$. If $|\epsilon(x)|$ and $|\eta(x)|$ are not equal to zero but are arbitrarily small, the numerator of the difference term will also be arbitrarily small. The denominator, however, will remain comparatively large since $\left| b^2\,a \int_c^d f_1(x)\,dx \int_c^d f_1(x)\,dx \right| \geqq b^2\,a\,B^2$, regardless of the arbitrarily small terms involving $\epsilon(x)$ and $\eta(x)$. Hence the difference term will be small for small values of $\epsilon(x)$ and $\eta(x)$; that is, for small variations in $k(x)$ and in the ratio of $f_2(x)$ to $f_1(x)$. A formal mathematical proof of the foregoing statements is not offered in this paper, which is intended primarily for engineers. The conclusions can be verified by well-known mathematical procedures.

## Classic Equations Applied to the Analysis of Two-Hinged Arches

The general equation for the horizontal reaction $H$ of a solid, two-hinged arch neglecting rib shortening and temperature effects is

$$H = \frac{\int M\,y\,\frac{ds}{E\,I}}{\int y^2\,\frac{ds}{E\,I}} \quad \ldots\ldots(8a)$$

in which $M$ = bending moment at any section due to given loads and vertical reactions only; $y$ = ordinate of the arch at any section; $ds$ = differential length of an element measured along the center line of the arch; $E$ = modulus of elasticity; and $I$ = rectangular moment of inertia of the arch section. In actual design, Eq. 8*a* is usually expressed as a summation, and the arch rib is divided into a finite number of segments of length $\Delta s$:

$$H = \frac{\sum M\,y\,\frac{\Delta s}{E\,I}}{\sum y^2\,\frac{\Delta s}{E\,I}} \quad \ldots\ldots(8b)$$

For two-hinged arch trusses:

$$H = \frac{\sum \frac{\sigma u L}{A E}}{\sum \frac{u^2 L}{A E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8c)$$

in which $\sigma$ is the total stress on the section due to given loads and vertical reactions only; $A$ is its area; $L$ is the length of the member; and $u$ is the total stress in the section due to a horizontal force of unity applied at the reaction.

It is presumed that Eqs. 8 are familiar to the reader. Their derivation can be found in texts on the theory of statically indeterminate structures.

The horizontal reaction is dependent upon the sizes of the sections or members of the arch. However, before the arch can be designed (that is, before the stresses that determine the size of sections or members can be computed) it is necessary to compute the horizontal reactions under various loads. Hence the sizes of sections or members must be assumed at the start and consequently the design is a process of successive approximations.

### Method of Reduced Equations Applied to Two-Hinged Arches

The purpose of the "method of reduced equations" is to eliminate as many steps as possible in the process of successive approximations and to provide a more direct solution. The method is so named because the application of the basic principle to the solution of two-hinged arches, and other structures, eliminates several quantities from the classic equations, thereby reducing them. In particular, the sizes of sections or members are eliminated so that the solution by this method depends only upon the center-line geometry of the structures.

*Definitions.*—Four new terms are introduced in this paper, as follows—(*a*) "difference term," (*b*) "reduced equation," (*c*) "basic influence line," and (*d*) "correspondence of ordinates."

(*a*) The "difference term" is the analytical expression of the difference between two fractions of the types shown in Eqs. 2 and 4.

(*b*) In the classical equation, $s$ is not necessarily the most convenient independent variable. Choosing $x$ as the independent variable, Eq. 8*a* can be written in terms of $x$:

$$\frac{\int \frac{M y\, ds}{E I}}{\int \frac{y^2\, ds}{E I}} = \frac{\int \frac{M y}{E I} \frac{ds}{dx} dx}{\int \frac{y^2}{E I} \frac{ds}{dx} dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

In general, $ds/dx = \sqrt{1 + (dy/dx)^2}$, which depends upon the form of the arch axis. If the cross section of the arch is so chosen as to make $E I$ proportional to $\sqrt{1 + (dy/dx)^2}$—then the quantity $\left(\frac{1}{E I}\frac{ds}{dx}\right)$ will be constant and can be factored from Eq. 9.

A "reduced equation" is the classic equation (Eq. 8*a*) with the quantity $\left(\frac{1}{E I}\frac{ds}{dx}\right)$, or $\frac{L}{A E}$, assumed constant and factored from the equation, thus:

For the solid type of arch,

$$H = \frac{\int M\, y\, dx}{\int y^2\, dx} \qquad (10a)$$

or, if $\Delta x$ is constant,

$$H = \frac{\sum M\, y}{\sum y^2} \qquad (10b)$$

and, for the arch truss,

$$H = \frac{\sum \sigma\, u}{\sum u^2} \qquad (11)$$

(*c*) A "basic influence line" is the one determined from the reduced equation of an arch.

(*d*) Two curves have "correspondence of ordinates" if the ratio of their ordinates at corresponding points on each curve remains approximately constant. For example, in Fig. 1(*a*), curves A and B have perfect correspondence of ordinates for their entire length; in Fig. 1(*b*), curves C and D have correspondence of ordinates, not perfect however, for their entire length; in Fig. 1(*c*), curves E and F have correspondence of ordinates only between 0 and point a; and, in Fig. 1(*d*), curves G and H have no correspondence of ordinates.

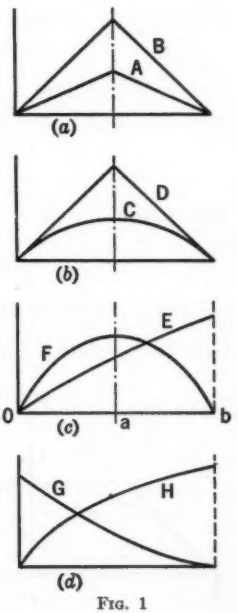


FIG. 1

The words "influence lines" and "influence values" throughout the discussion of two-hinged arches are understood to refer always to the horizontal reactions.

*Procedure.*—The use of the "reduced equations" (Eqs. 10 or 11) in the design of two-hinged arches is worth while when these equations give values sufficiently close for practical purposes to the final correct values determined from the classic equations (Eqs. 8).

The following procedure will be followed in applying the method of reduced equations to the solution of solid-type, two-hinged arches: First, the effect of variations in cross section upon the basic influence line will be studied for parabolic, circular, and semi-elliptic arches successively; and second, the effect of variation in curve or geometry of the arch upon the influence lines will be studied.

## EFFECT OF VARIATION IN CROSS SECTION

*General.*—The difference between the two fractions in Eq. 2 is zero when $\frac{a_1}{b_1} = \frac{a_2}{b_2}$, regardless of the values of $k_1$ and $k_2$. Similarly (compare Eq. 9), the

difference between $\dfrac{\sum_1^n M\,y}{\sum_1^n y^2} - \dfrac{\sum_1^n M\,y\,\dfrac{\Delta s}{E\,I}}{\sum_1^n y^2\,\dfrac{\Delta s}{E\,I}}$ is zero, when $\dfrac{M_1\,y_1}{y^2_1} = \dfrac{M_2\,y_2}{y^2_2} = \cdots = \dfrac{M_n\,y_n}{y^2_n}$, regardless of the $\left(\dfrac{\Delta s}{E\,I}\right)$-values.

If the $\left(\dfrac{M\,y}{y^2}\right)$-values are not all equal, but nearly so, then the difference between the two fractions will be very small, regardless of the $\left(\dfrac{\Delta s}{E\,I}\right)$-values.

In terms of the integral equations, the difference $\dfrac{\int M\,y\,dx}{\int y^2\,dx} - \dfrac{\int M\,y\,\dfrac{ds}{E\,I}}{\int y^2\,\dfrac{ds}{E\,I}}$ will be zero or very small, if the ratio $\dfrac{M\,y}{y^2}$ is constant, or nearly so, for all values of $x$.

*Parabolic Arch.*—The equation of a parabolic arch with the origin at the left end, with span, $L$, and rise, $h$, is

$$y = \frac{4\,h}{L^2}\,(x\,L - x^2) \qquad (12a)$$

and letting $l = L/2$,

$$y = \frac{h}{l^2}\,(2\,l\,x - x^2) \qquad (12b)$$

For a parabolic arch with a unit load at the center, and $\dfrac{L}{h} = 4$, characteristic curves for $M$-values, $y$-values, $(M\,y)$-values, and $y^2$-values are shown in

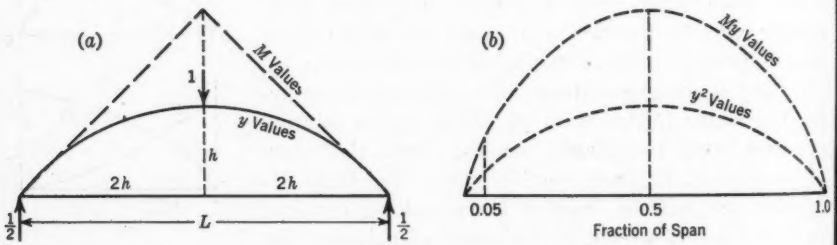



FIG. 2

Fig. 2. For different values of $x$, the moment $M$ is as follows: When $0 \leqq x \leqq l$,

$$M = 0.5\,x \qquad (13a)$$

and, when $l \leqq x \leqq L$,

$$M = l - 0.5\,x \qquad (13b)$$

Values of $H = \dfrac{M\,y}{y^2}$ (Eq. 10*b*) are given in Table 1 for $l/h = 2$, 4, and 8.

From Fig. 2, it is seen that there is "correspondence of ordinates" between the curves representing ($M\,y$)-values and $y^2$-values and that the difference term for a parabolic arch with center load will be small, regardless of the variations in cross sections or sizes of members.

For intermediate positions of the unit load, there is not correspondence of ordinates for the entire span length. In Fig. 3, the curves of $M$-values and $y$-values do not correspond between points B and C. However, the error introduced by the small lack of correspondence between points B and C is of opposite sign from the error introduced by imperfection of correspondence between points A and B and between points C and D, so that at one position of the intermediate load, there is no error. It must be kept in mind that the errors mentioned are those between the basic influence line determined from Eq. 10*a* and the actual influence line determined from Eq. 8*a*.

TABLE 1.—VALUES OF $\frac{M\,y}{y^2}$ (Eq. 10*b*)

| Fraction of span[a] | VALUES OF $L/h$ | | |
|---|---|---|---|
| | 2 | 4 | 8 |
| 0.05 | 0.26 | 0.52 | 1.04 |
| 0.50 | 0.50 | 1.00 | 2.00 |

[a] See Fig. 2(*a*).

The correctness of the conclusion that variations in cross section will have but little effect on the influence values because of the correspondence of ordinates is shown by the following computations of the basic influence line

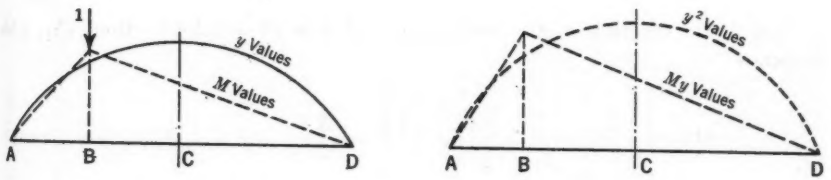



FIG. 3

and influence lines for several types of cross section. These computations also reveal the fact that the basic influence lines for the parabolic arch are directly proportional to the span: rise ratio. The basic influence line for the parabola is computed from Eq. 10*a* between the limits of 0 and $L$. By using a double unit load, placed symmetrically, it is necessary to integrate only over one half of the span, since both the arch and the moment diagram are symmetrical about the center line. Thus (see Fig. 4):

$$2\,H = \frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx}{\int_0^l y^2\,dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

or

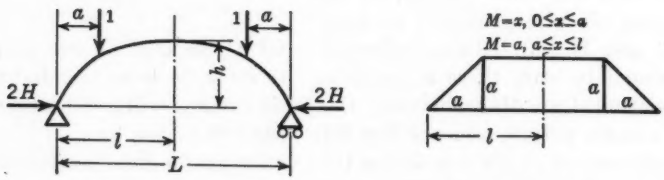
$$H = \frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx}{\int_0^L y^2\,dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

In Eq. 14*b*, substituting the value of $y$ from Eq. 12*b*:

$$H = \frac{\frac{h}{l^2}\int_0^a (2\,l\,x^2 - x^3)\,dx + \frac{a\,h}{l^2}\int_a^l (2\,l\,x - x^2)\,dx}{\frac{h^2}{l^4}\int_0^L (2\,l\,x - x^2)^2\,dx} \quad \ldots\ldots (15a)$$

which becomes

$$H = \frac{\frac{h}{l^2}\left[\frac{2\,l\,x^3}{3} - \frac{x^4}{4}\right]_0^a + \frac{a\,h}{l^2}\left[l\,x^2 - \frac{x^3}{3}\right]_a^l}{\frac{h^2}{l^4}\left[\frac{4\,l^2\,x^3}{3} - \frac{4\,l\,x^4}{4} + \frac{x^5}{5}\right]_0^L} = \frac{\frac{l^2}{h}\left[\frac{2\,l^3\,a}{3} - \frac{l\,a^3}{3} + \frac{a^4}{12}\right]}{\frac{4\,l^2\,L^3}{3} - l\,L^4 + \frac{L^5}{5}} \quad \ldots\ldots (15b)$$




(*a*) LOAD DIAGRAM (*b*) MOMENT DIAGRAM

FIG. 4

Let $j$ = a fraction of the half span, and $a = j\,l = 0.5\,j\,L$; then Eq. 15*b* becomes

$$H = \frac{\frac{1}{h}\left(\frac{L}{2}\right)^6\left[\frac{2\,j}{3} - \frac{j^3}{3} + \frac{j^4}{12}\right]}{\frac{L^5}{3} - \frac{L^5}{2} + \frac{L^5}{5}} = \frac{5\,L}{128\,h}(8\,j - 4\,j^3 + j^4) \quad \ldots\ldots (16a)$$

For a unit load at the center, $j = 1$ and

$$H = \frac{25\,L}{128\,h} = 0.1953\,\frac{L}{h} \quad \ldots\ldots (16b)$$

It is observed that the basic influence line is a direct function of the $(L/h)$-ratio. Consequently, for basic influence lines, one curve can be plotted for all parabolic arches, as a function of that ratio. Influence lines for non-constant $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-values are computed from Eq. 8*a* (between the limits 0 and $l$). In this paper, influence lines are computed first for six types of arch rib which were selected primarily because of their mathematical simplicity. These are shown in Fig. 5(*b*). Also shown in Fig. 5(*b*) is the diagram for an arch with a constant quantity $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$. For each type the quantity $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$ is expressed as a function of $x$ in Col. 2, Table 2. Types 1, 2, 4, and 5 represent functions which are not symmetrical about the center line of the arch, and can

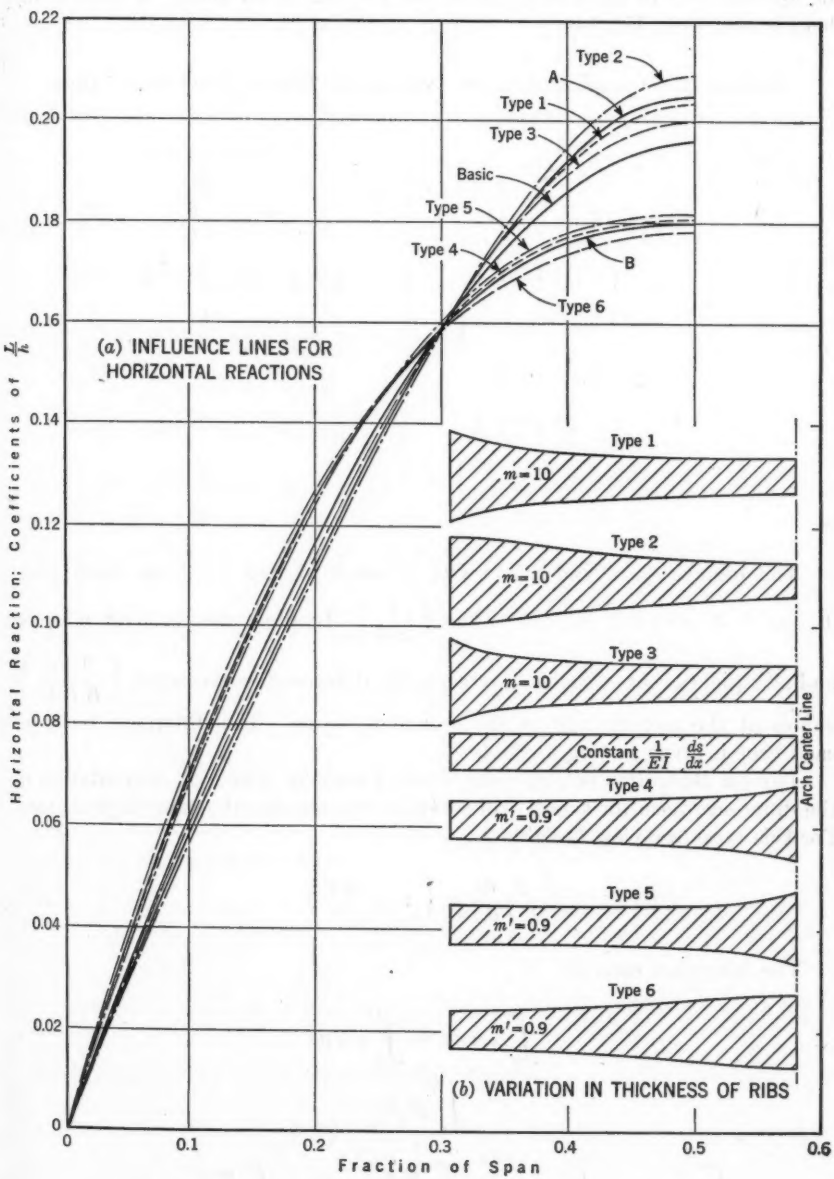



FIG. 5.—PARABOLIC ARCH RIBS

be applied only in integrations over the left half of an arch. A double unit load is used as in Eq. 14*a*.

TABLE 2.—COMPUTATION OF INFLUENCE LINES, PARABOLIC ARCH

| Type (see Fig. 5) (1) | $\frac{1}{E\,I}\frac{ds}{dx}$ (2) | Difference term (3) |
|---|---|---|
| 1 | $\left[1 + m\,\frac{x}{l}\right]$ | $\frac{m}{128\,(16 + 11\,m)}\,\frac{L}{h}\,(55\,g_1 - 16\,g_2)$ |
| 2 | $\left[1 + m\left(\frac{x}{l}\right)^2\right]$ | $\frac{m}{128\,(56 + 29\,m)}\,\frac{L}{h}\,(145\,g_1 - 112\,g_3)$ |
| 3 | $\left[1 + m\,\frac{(2\,l\,x - x^2)}{l^2}\right]$ | $\frac{m}{64\,(7 + 6\,m)}\,\frac{L}{h}\,(15\,g_1 - 7\,g_4)$ |
| 4 | $\left[1 - m'\,\frac{x}{l}\right]$ | $\frac{m'}{128\,(16 - 11\,m')}\,\frac{L}{h}\,(16\,g_2 - 55\,g_1)$ |
| 5 | $\left[1 - m'\left(\frac{x}{l}\right)^2\right]$ | $\frac{m'}{128\,(56 - 29\,m')}\,\frac{L}{h}\,(112\,g_3 - 145\,g_1)$ |
| 6 | $\left[1 - m'\,\frac{(2\,l\,x - x^2)}{l^2}\right]$ | $\frac{m'}{64\,(7 - 6\,m')}\,\frac{L}{h}\,(7\,g_4 - 15\,g_1)$ |

The quantities $m$ and $m'$ in Col. 2 are arbitrary constants such that: $0 \leqq m < \infty$ and $0 \leqq m' < 1$. The $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-values are constant when $m$ and $m'$ equal 0. As $m$ and $m'$ increase, the difference between the $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-values at the haunch and at the crown increases. The difference term for each type is given in Col. 3, Table 2.

*Typical Derivation of Difference Term, Parabolic Arch.*—A computation of the difference term for type 1 will serve to demonstrate the general procedure. For this type (see Col. 2, Table 2),

$$\frac{1}{E\,I}\frac{ds}{dx} = \left(1 + \frac{m\,x}{l}\right) \qquad (17)$$

The difference term is:

$$\frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx}{\int_0^L y^2\,dx}$$

$$- \frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx + \int_0^a \frac{m\,x}{l}\,x\,y\,dx + \int_a^l \frac{m\,x}{l}\,a\,y\,dx}{\int_0^L y^2\,dx + 2\int_0^l \frac{m\,x}{l}\,y^2\,dx} = G \qquad (18)$$

From the foregoing derivation (Eqs. 14*a* and 15*b*):

$$\int_0^a x\,y\,dx + \int_a^l a\,y\,dx = \frac{h\,l^2}{12}(8\,j - 4\,j^3 + j^4) \ldots\ldots\ldots (19a)$$

and, letting $g_1 = 8\,j - 4\,j^3 + j^4$,

$$\int_0^a x\,y\,dx + \int_0^l a\,y\,dx = \frac{h\,l^2}{12}\,g_1 = \frac{h\,L^2}{48}\,g_1 \ldots\ldots\ldots\ldots (19b)$$

Also,

$$\int_0^L y^2\,dx = \frac{h^2}{l^4} \times \frac{L^5}{30} = \frac{16}{30}\,h^2\,L \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The two expressions involving $m$ in the numerator of Eq. 18 are now evaluated:

$$\int_0^a \frac{m\,x}{l}\,x\,y\,dx + \int_a^l \frac{m\,x}{l}\,a\,y\,dx = \int_0^a \frac{m\,h}{l^3}(2\,l\,x^3 - x^4)\,dx$$

$$+ \int_a^l \frac{m\,h\,a}{l^3}(2\,l\,x^2 - x^3)\,dx = \frac{m\,h}{l^3}\left[\frac{2\,l\,x^4}{4} - \frac{x^5}{5}\right]_0^a \frac{m\,h\,a}{l^3}\left[\frac{2\,l\,x^3}{3} - \frac{x^4}{4}\right]_a^l$$

$$= \frac{m\,h}{l^3}\left[\frac{l\,a^4}{2} - \frac{a^5}{5}\right] + \frac{m\,h}{l^3}\left[\frac{2\,l^4\,a}{3} - \frac{l^4\,a}{4} - \frac{2\,l\,a^4}{3} + \frac{a^5}{4}\right] \ldots\ldots (21)$$

Substituting $a = j\,l$, in Eq. 21:

$$\int_0^a \frac{m\,x}{l}\,x\,y\,dx + \int_a^l \frac{m\,x}{l}\,a\,y\,dx = m\,h\,l^2\left[\frac{j^4}{2} - \frac{j^5}{5} + \frac{2\,j}{3} - \frac{j}{4} - \frac{2\,j^4}{3} + \frac{j^5}{4}\right]$$

$$= \frac{m\,h\,l^2}{60}(25\,j - 10\,j^4 + 3\,j^5) = \frac{m\,h\,L^2}{240}\,g_2 \ldots\ldots\ldots\ldots (22)$$

in which $g_2 = 25\,j - 10\,j^4 + 3\,j^5$. The term involving $m$ in the denominator of Eq. 18 is next evaluated; thus:

$$2\int_0^l \frac{m\,x}{l}\,y^2\,dx = \frac{2\,m}{l}\int_0^l x\,\frac{h^2}{l^4}(2\,l\,x - x^2)^2\,dx$$

$$= \frac{2\,m\,h^2}{l^5}\int_0^l (4\,l^2\,x^3 - 4\,l\,x^4 + x^5)\,dx = \frac{11}{30}\,m\,h^2\,L \ldots\ldots\ldots (23)$$

Evaluating the difference term (see Eq. 6) gives

$$G = \frac{\dfrac{h\,L^2\,g_1}{48}\dfrac{11\,m\,h^2\,L}{30} - \dfrac{m\,h\,L^2\,g_2}{240}\dfrac{16\,h^2\,L}{30}}{\dfrac{16\,h^2\,L}{30}\left(\dfrac{16\,h^2\,L}{30} + \dfrac{11\,m\,h^2\,L}{30}\right)}$$

$$= \frac{m}{128\,(16 + 11\,m)}\,\frac{L}{h}\,(55\,g_1 - 16\,g_2) \ldots\ldots\ldots\ldots (24)$$

The substitution constants $g$ in Col. 3, Table 2, are as follows:

$$g_1 = (8j - 4j^3 + j^4) \quad \ldots\ldots (25a)$$

$$g_2 = (25j - 10j^4 + 3j^5) \quad \ldots\ldots (25b)$$

$$g_3 = (9j - 3j^5 + j^6) \quad \ldots\ldots (25c)$$

$$g_4 = (16j - 10j^4 + 6j^5 - j^6) \quad \ldots\ldots (25d)$$

An examination of the difference terms shows that the errors are also a direct function of the $(L/h)$-ratios. Consequently, the influence lines for any parabolic arch of any of these types of $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-values can be obtained from one curve of that type, plotted in terms of $L/h$. The influence lines for these six types of arch sections are plotted with the basic influence line in Fig. 5(*a*). Values of $m = 10$ and $m' = 0.9$ were used.

The influence values are not greatly affected by variations in the ratio of haunch thickness to crown thickness. Different values of this ratio are obtained by using different values of $m$ or $m'$ in the expressions for the cross sections of types 1 to 6 (Col. 2, Table 2). Since the influence values are proportional to the $(L/h)$-ratio, illustrative computations are shown only for a specific arch of $L/h = 4$, the computations for any other parabolic arch being directly proportional to these. The maximum influence ordinate at the crown for the basic influence line of a parabolic arch of $L/h = 4$ is 0.781. Tables 3 and 4 show the amounts by which the maximum influence ordinates for the

TABLE 3.—INFLUENCE ORDINATES FOR PARABOLIC ARCH TYPES 1 TO 3—CROSS SECTIONS DECREASING FROM THE END TO THE CENTER $\left(\text{USING } m\text{-CONSTANTS AND } \frac{L}{h} = 4\right)$ WITH A UNIT LOAD AT THE CROWN

| Constant $m$ | Ratio of haunch thickness to crown thickness | TYPE 1 | | TYPE 2 | | TYPE 3 | |
|---|---|---|---|---|---|---|---|
| | | Difference term | Ordinate | Difference term | Ordinate | Difference term | Ordinate |
| 5 | 2.0 : 1.0 | +0.029 | 0.810 | +0.046 | 0.827 | +0.017 | 0.798 |
| 10 | 2.5 : 1.0 | +0.032 | 0.813 | +0.053 | 0.834 | +0.019 | 0.800 |
| 20 | 3.1 : 1.0 | +0.034 | 0.815 | +0.058 | 0.839 | +0.020 | 0.801 |
| 40 | 3.9 : 1.0 | +0.036 | 0.817 | +0.061 | 0.842 | +0.020 | 0.801 |

six types of cross sections differ from this value. It is observed that increasing the haunch thickness from twice to four times the crown thickness produces a maximum change in ordinate, for types 1, 2, and 3, of $0.061 - 0.046 = 0.015$. Increasing the crown thickness from one and a half to three times the haunch thickness produces a maximum change, for types 4, 5, and 6, of $0.106 - 0.045 = 0.061$. These effects are shown graphically in Fig. 5(*a*).

Influence values for a unit load at the center only were computed for several types of cross-section distributions, with thicknesses selected arbitrarily

and shown in Fig. 6 as types 7 to 11. These types were selected as distributions not represented in types 1 to 6. The column analogy method was used for these computations, since the mathematical expressions of $\frac{1}{E\,I}\frac{ds}{dx}$ for these types are complicated. Results are shown in Fig. 7.

TABLE 4.—INFLUENCE ORDINATES FOR PARABOLIC ARCH TYPES 4 TO 6—CROSS SECTIONS INCREASING FROM THE END TO THE CENTER $\left(\text{USING } m'\text{-CONSTANTS AND } \frac{L}{h} = 4\right)$ WITH A UNIT LOAD AT THE CROWN

| Constant $m'$ | Ratio of haunch thickness to crown thickness | Type 4 | | Type 5 | | Type 6 | |
|---|---|---|---|---|---|---|---|
| | | Difference term | Ordinate | Difference term | Ordinate | Difference term | Ordinate |
| 0.8 | 1.0 : 1.5 | −0.045 | 0.736 | −0.045 | 0.736 | −0.045 | 0.736 |
| 0.9 | 1.0 : 1.9 | −0.060 | 0.721 | −0.055 | 0.726 | −0.070 | 0.711 |
| 0.95 | 1.0 : 2.4 | −0.070 | 0.711 | −0.062 | 0.719 | −0.091 | 0.690 |
| 0.975 | 1.0 : 3.1 | −0.075 | 0.706 | −0.065 | 0.716 | −0.106 | 0.675 |

Inspection of the data shows that the influence lines fall in three groups—the basic influence line, a group of influence lines for diminishing cross section from end to center of arch, and a group for increasing cross section from end to center of arch.

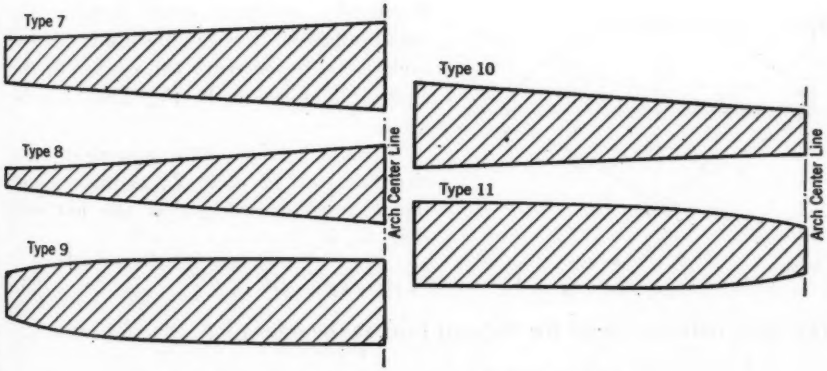



FIG. 6.—VARIATION IN THICKNESS OF PARABOLIC ARCH RIBS

Influence lines for the parabola can be generalized by plotting an average curve A for the group of influence lines above the basic influence line and an average curve B for the group below the basic influence line, as are shown in Fig. 5(*a*). The maximum percentage error encountered in using the average curve in each group is approximately 2% for the top group (cross section

diminishing toward center) and approximately 6% for the bottom group (cross section increasing toward center), for a maximum ratio between thicknesses at crown and haunch of about 4.5:1, which is very high. Thus, if the arch is to

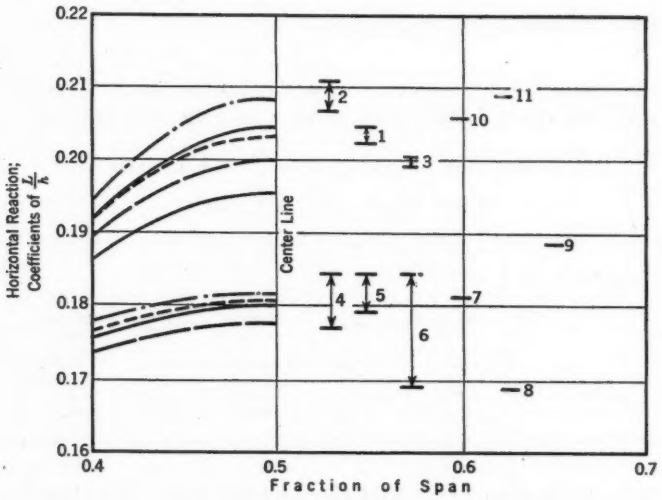



FIG. 7.—RANGES OF INFLUENCE VALUES FOR TYPES 1 TO 6 AND MAXIMUM VALUES FOR TYPES 7 TO 11

have haunches, influence line A, Fig. 5(*a*), can be used to make a direct solution for the reactions and required cross-section dimensions. If the arch is to have practically constant cross section, the basic influence line can be used. If the arch is to taper from the center to the ends, influence line B, Fig. 5(*a*), can be used.

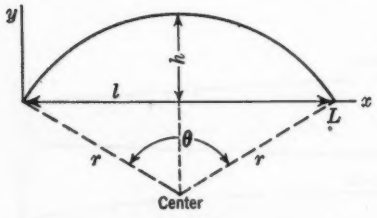



FIG. 8

*Circular Arch.*—The equation of a circular arch of span $L$, radius $r$, and rise $h$, with the origin at the left end (Fig. 8), is

$$y = h - r + \sqrt{r^2 - (x - l)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

The basic influence value for the unit load at the crown is:

$$H = \frac{\int_0^l x\, y\, dx}{\int_0^L y^2\, dx} = \frac{3\,l\,r^2\,\theta + 6\,h\,r\,(h - r) - 2\,h^3}{12\,(h - r)\,r^2\,\theta + 12\,l\,r^2 - 4\,l^3} \quad \ldots\ldots\ldots\ldots (27)$$

The influence values (see Table 5, lines 1 and 2) are not a linear function of $L/h$, as in the case of the parabola.

Basic influence lines for $(L/h)$-ratios of 8, 4, and 2 were computed by column analogy. Influence lines for variations in $\frac{1}{E\,I}\frac{ds}{dx}$ of types 1 to 6 were

FIG. 9.—INFLUENCE LINES FOR HORIZONTAL REACTION, SEMICIRCULAR ARCH RIBS, WHEN A UNIT LOAD IS AT THE CENTER OF THE SPAN

also computed by this method. The results are shown in Fig. 9. The variations from the basic influence line increase as the $(L/h)$-ratio decreases—that is, as the curve of the arch departs more and more from a straight line.

TABLE 5.—BASIC INFLUENCE VALUES AT THE CENTER OF SEMICIRCULAR ARCHES, WITH A UNIT LOAD AT THE CROWN

| No. | Description | Ratio of Span Length $L$ to Rise $h$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 14 : 1 | 12 : 1 | 10 : 1 | 8 : 1 | 6 : 1 | 4 : 1 | 3 : 1 | 2.5 : 1 | 2 : 1[a] |
| | Basic Values: | | | | | | | | | |
| 1 | Actual | 2.7240 | 2.3337 | 1.9404 | 1.5466 | 1.1512 | 0.752 | 0.548 | 0.4439 | 0.339 |
| 2 | In terms of $L : h$ | 0.1946 | 0.1945 | 0.1940 | 0.1933 | 0.1919 | 0.1880 | 0.1828 | 0.1775 | 0.1695 |
| 3 | Ratio[b] | 0.9964 | 0.9959 | 0.9933 | 0.9898 | 0.9826 | 0.9626 | 0.9360 | 0.9089 | 0.8679 |

[a] Semicircle or semi-ellipse.

[b] $\dfrac{\text{Maximum coefficient for a circle}}{\text{Maximum coefficient for a parabola (that is, 0.1953)}}$.

*Semi-Ellipse.*—The equation of the semi-ellipse with the origin at the left end is

$$y = \frac{h}{l}\sqrt{2\,l\,x - x^2} \qquad (28)$$

For a unit load at the crown, the basic influence ordinate is

$$H = \frac{\int_0^l x\,y\,dx}{\int_0^L y^2\,dx} = \frac{3\,\pi - 4}{32}\frac{L}{h} = 0.1695\,\frac{L}{h} \qquad (29)$$

In the case of the semi-ellipse, as for the parabola, the influence ordinates are found to be a linear function of the $(L/h)$-ratio. The basic influence values are also proportional to those for a semicircle $(L/h = 2)$. The basic influence line for the semi-ellipse and influence lines for types 1 to 6 were computed by column analogy and are similar to those for a circle of $L/h = 2$ (see Fig. 9(c)).

## EFFECT OF VARIATIONS IN CURVE

The comparative curves of the parabolic, circular, and semi-elliptic arches are shown in Fig. 10. An examination of these curves, together with the respective influence lines in Fig. 9 and the data in Table 5, shows that the more the curve departs from a straight line—(1) the lower are the influence values, and (2) the greater is the effect of variation in cross section upon these values.

## GENERALIZED CURVES FOR TWO-HINGED ARCHES

Fig. 11 is a comparative diagram of the basic influence lines for the parabolic, circular, and semi-elliptical arch, plotted as fractions of the maximum values. It shows that, for practical purposes, all the basic influence lines have the same relative shape. This is also true for the influence lines for different $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-variations. Consequently, using the parabola as a standard, an

influence line for any circular or semi-elliptic arch can be determined by multiplying the influence values for the corresponding parabola (basic, A, or B in Fig. 5(*a*)) by the ratio of the respective ($L/h$)-coefficients. Table 5, line 3, gives the ratios of coefficients for basic influence values.

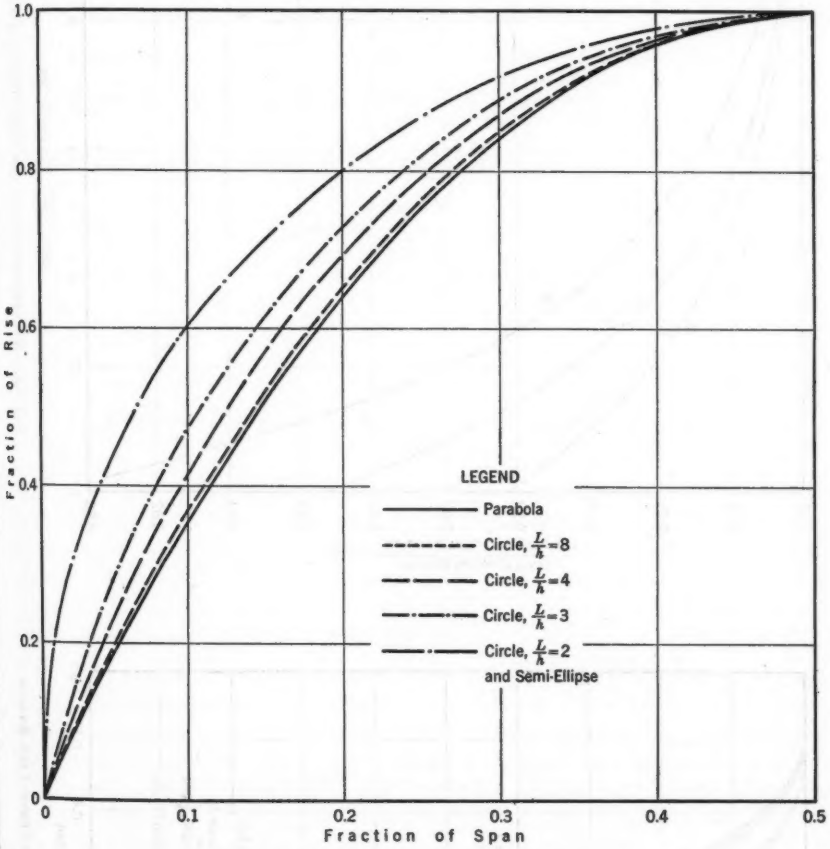



FIG. 10.—COMPARISON OF CURVE SHAPES

Plotting these points gives a reference curve for general use in the solid line in Fig. 12. This is used in conjunction with the basic values for the parabola in Fig. 5(*a*). Thus, for a circular arch of $L/h = 3.5$, the basic influence ordinates are obtained by multiplying the basic influence ordinates of the parabola (Fig. 5(*a*)) by $3.5 \times 0.950 = 3.325$, the factor 0.950 being taken from the basic reference curve in Fig. 12. Two similar average reference curves for the average curves A and B are plotted in Fig. 12 as dotted lines. For other arch forms, such as a multi-centered arch, segment of an ellipse, and others, the comparative curve form can be plotted in Fig. 10, and influence values determined from that curve which most nearly approximates it.

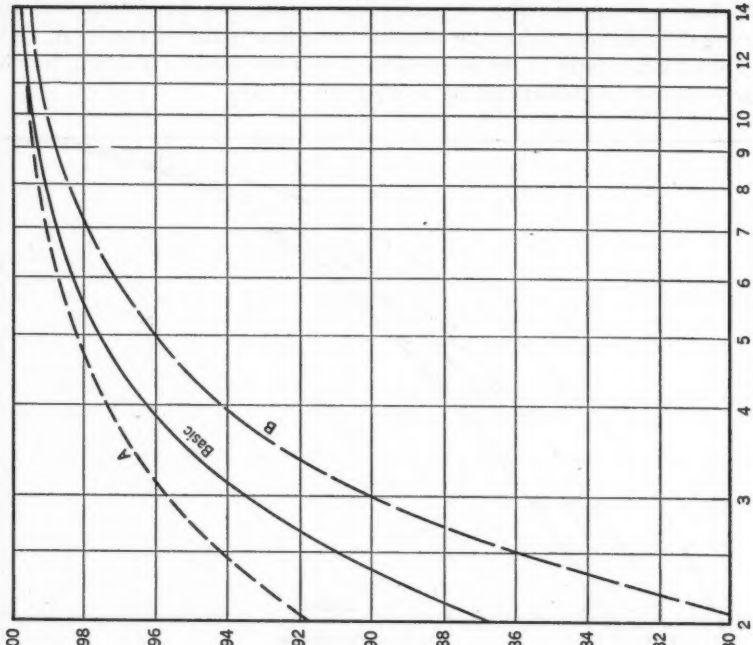



FIG. 12.—REFERENCE CURVES FOR TWO-HINGED ARCHES

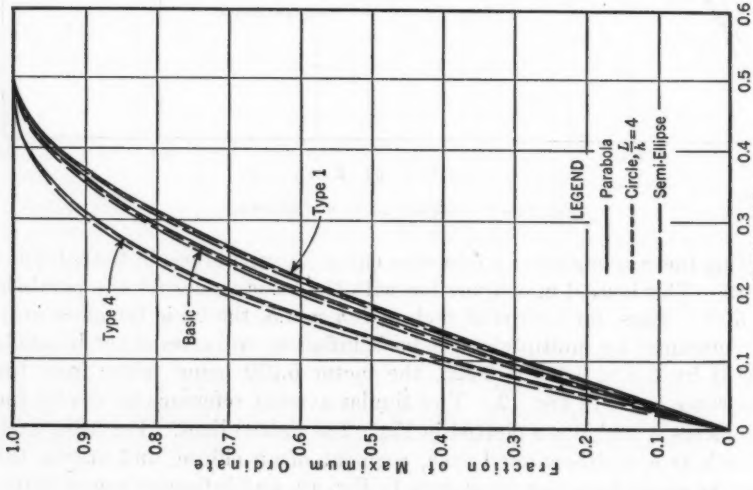



FIG. 11.—COMPARISON OF INFLUENCE LINE SHAPES

## Arch Trusses

The reference curves in this paper apply equally well to two-hinged arch trusses. Computations show that the relation between the reduced equation $\frac{\sum \sigma u}{\sum u^2}$ and the classic equation $\frac{\sum \frac{\sigma u L}{A E}}{\sum \frac{u^2 L}{A E}}$ for truss structures is substantially the same as for solid structures.[2] Illustrative data are presented for three framed arches in Table 6. The Tyngsborough arch is parabolic in shape and of constant depth, corresponding to constant $\left(\frac{1}{E I}\frac{ds}{dx}\right)$-values. The Hell Gate and the Cottage Farm arches both have parabolic lower chords and are haunched in shape, the minimum depth being at the crown. The rise in each case is from the hinges to the midpoint between top and bottom chords at the crown. The Hell Gate and Cottage Farm arches are taken as parabolic in Table 6.

TABLE 6.—Comparative Data for Three Two-Hinged Arches

| No. | Arch | Location | Type | Ratio, span to rise | Maximum Influence Ordinate | |
|---|---|---|---|---|---|---|
| | | | | | "Reduced" | Actual |
| 1 | Tyngsborough | Lowell, Mass. | Basic | 5.71 | 1.113 | 1.104 |
| 2 | Cottage Farm | Boston, Mass. | A | 4.165 | 0.852 | 0.838 |
| 3 | Hell Gate | New York, N. Y. | A | 4.16 | 0.849 | 0.841 |

## Limitations

The accuracy of the reference curves decreases as the arch form becomes more curved, the errors varying from approximately 1% to 3% for the parabola to approximately 5% to 8% for the semi-ellipse, with usual arch shapes that do not have excessive differences in thickness between crown and haunch.

These reference curves will not apply to rigid-framed bents, and other similar structures, solved as two-hinged arches, because of their near-rectangular shape and consequent lack of "correspondence of ordinates." A special study of these shapes is necessary to establish the relationship between the geometry of the bents and the large errors resulting from reducing the equations.

## Conclusion

The purpose of this paper has been to state a principle and to illustrate its application, but not to present an exhaustive treatise. Consequently the following considerations and many more have been omitted intentionally: Initial selection of the curve and shape of the arch rib or arch truss; rare arch forms of curves falling between a parabola and a straight line; rigid-frame

[2] "A Simplified Method for Solving Statically Indeterminate Framed Structures," by Lee H. Johnson, Jr., presented to Harvard Univ., Cambridge, Mass., in May, 1935, in partial fulfilment of the requirements for the degree of Doctor of Science in Civil Engineering.

bents and rectangular two-hinged structures; rib shortening; temperature effects; the effect of dissymmetry; and structures statically indeterminate to the second degree and higher. Furthermore, the study of the effect of variations in the thickness of the arch rib has been restricted to a small group of representative sections which is deemed sufficient for the practical application of the results. There would be no end to a "complete" investigation of this effect since there is an infinite number of possible variations.

The basic principle is applicable, however, to the analysis of certain highly indeterminate structures. A simple example is a three-span continuous truss, statically indeterminate to the second degree. The classic equations for the indeterminate reactions, $R_1$ and $R_2$, can be written:

$$R_1 \sum \frac{u^2_1 L}{A E} + R_2 \sum \frac{u_1 u_2 L}{A E} + \sum \frac{\sigma u_1 L}{A E} = 0 \ldots\ldots\ldots (30a)$$

$$R_1 \sum \frac{u_1 u_2 L}{A E} + R_2 \sum \frac{u^2_2 L}{A E} + \sum \frac{\sigma u_2 L}{A E} = 0 \ldots\ldots\ldots (30b)$$

The reduced equations would be:

$$R_1 \sum u^2_1 + R_2 \sum u_1 u_2 + \sum \sigma u_1 = 0 \ldots\ldots\ldots\ldots (31a)$$

$$R_1 \sum u_1 u_2 + R_2 \sum u^2_2 + \sum \sigma u_2 = 0 \ldots\ldots\ldots\ldots (31b)$$

Computations of continuous trusses in a paper by Albert Haertlein,[3] M. Am. Soc. C. E., show good agreement between the results obtained from the two sets of equations.

The "method of reduced equations" can be used to analyze structures statically indeterminate to the first degree where there is "correspondence of ordinates." It reduces the labor required of the designer and makes possible quicker solutions.

The factors governing the application of the method to the analysis of more highly indeterminate structures are not known to the writer. The development of such applications would be of considerable value to the designing engineer. The few computations the writer has made indicate that, if small deformations or movements of a structure produce large changes in any of its reactions, the method of reduced equations will not necessarily give close approximations to those reactions. The moments at the ends of a stiff, fixed-end arch are such reactions.

## APPENDIX

### NOTATION

The following letter symbols, used in this paper, conform essentially to American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[4] prepared by a Committee of the American Standards Association and approved by the Association in 1932:

[3] "The Design of Statically Indeterminate Trusses," by Albert Haertlein, *Journal*, Boston Soc. of Civ. Engrs., April, 1936, p. 57.

[4] ASA—Z10a—1932.

$A$ = area of the cross section of the arch rib;
$a$ = coefficient; the letters $a$, $b$, $B$, $c$, and $d$ are used as mathematical coefficients, or arbitrary constants, in the introduction part of this paper (see "Basic Principle"); also $a$ = distance along the span from the nearest support (see Fig. 4);
$B$ = an arbitrary constant (see $a$);
$c$ = an arbitrary constant (see $a$);
$d$ = an arbitrary constant (see $a$);
$E$ = modulus of elasticity;
$f$ = "function of," distinguished by numerical subscripts as in Eq. 4; see also $\epsilon$ and $\eta$;
$G$ = difference term as defined by Eqs. 2 and 6;
$g$ = substitution factors $g_1$, $g_2$, $g_3$, and $g_4$, defined by Eqs. 25;
$H$ = horizontal reaction;
$h$ = rise of the arch;
$I$ = moment of inertia of the cross section of the arch rib;
$j$ = a fraction of the half span $l = a/l$ (see Eq. 16$a$);
$k$ = mathematical multipliers $k_1$, $k_2$, $\cdots$, $k_n$, used to demonstrate the basic principle (see Eq. 1);
$L$ = span length of arch axis = $2\,l$;
$l$ = half the span length = $L/2$;
$M$ = bending moment due to given loads and vertical reactions only;
$m$ = arbitrary constant in the difference terms (Table 2, Col. 3) for arches that taper downward from the support to the center; $m'$ = an arbitrary constant for arches that taper;
$n$ = number of terms, as in Eq. 3;
$R$ = reactions (see Eqs. 30);
$s$ = length of an arch measured along its center line axis; $\Delta s$ = length of a small segment of arc;
$u$ = total stress in an arch member due to a horizontal force of unity applied at the reaction;
$x$ = abscissa distance to any point $x,y$;
$y$ = ordinate distance to any point $x,y$; the rise of the arch at a distance $x$ from the left support;
$\epsilon$ = "function of"; $\epsilon(x)$ = a function whose absolute value can be made arbitrarily small; see also $f$ and $\eta$;
$\eta$ = "function of"; see $f$ and $\epsilon$;
$\sigma$ = total stress in an arch member due to given loads and vertical reactions only.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

# Principles of Depreciation

FINAL REPORT OF THE SPECIAL COMMITTEE
AUTHORIZED BY THE BOARD OF DIRECTION
TO ANALYZE AND DISCUSS THE
1943 REPORT OF THE NATIONAL ASSOCIATION OF
RAILROAD AND UTILITIES COMMISSIONERS'
COMMITTEE ON DEPRECIATION

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COMMITTEE

HENRY E. RIGGS, *Chairman*

T. R. AGG N. T. VEATCH, JR. L. R. HOWSON

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SEPTEMBER 30, 1944

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# PRINCIPLES OF DEPRECIATION

## FINAL REPORT OF THE SPECIAL COMMITTEE AUTHORIZED BY THE BOARD OF DIRECTION TO ANALYZE AND DISCUSS THE 1943 REPORT OF THE NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS' COMMITTEE ON DEPRECIATION

### I.—Introduction

The Special Committee of the Society on Depreciation respectfully submits this, its final report on the subject of "Depreciation," for the purpose of expanding the discussion of important phases of the subject which could only be suggested, and the committee's opinion stated, in the Progress Report submitted[1] in April, 1944. The limited time then available to the committee made it impossible to secure records of past experience of any of the different utilities which have built up reserves for "depreciation" under regulated accounting, or for the members of the committee to discuss thoroughly the many different and complex phases of the subject so as to be in accord on an extended discussion of this most important subject.

The Report of the Committee on Depreciation of the National Association of Railroad and Utilities Commissioners (hereinafter referred to as the NARUC committee) is devoted primarily to a discussion of methods to be used in a revision of the Accounting Classification to be required. By far the largest part of that report is devoted to the purely accounting phase of the subject and to detailed discussions of various methods—straight-line, sinking fund, compound interest, and other methods, problems that belong primarily to the Accounting Profession. There are many things in that report with which the Society's committee is in agreement. The Society's committee in this, as in the Progress Report, has limited its discussion to the fundamental differences of opinion which have developed from a study of the NARUC committee's report.

That report, when analyzed, recommends that annual depreciation expense be calculated by the straight-line method of distribution, using as a basis for the distribution the average lives of groups or units of property. Also, it recommends that the straight-line method be used for the computation of the de-

[1] *Proceedings*, Am. Soc. C. E., June, 1944, p. 887.

preciation reserve and that such reserve, when so computed, shall measure the actual depreciation in the property.

Even when the most complete and accurate data on average life are available, the straight-line method of distribution is still open to the objection that it assumes that a unit of service rendered today and one to be rendered some years in the future are of equal value, which obviously they are not.

The Society's committee, while fully recognizing the usefulness of the straight-line method in accounting for unregulated industrial property and possibly for certain utility properties, does not believe that its use is proper in the case of the great majority of utility properties; or that the reserve thus computed represents the "actual depreciation"; or that the depreciation reserve accumulated under regulated accounting should be "corrected" or readjusted to make it conform to the amount computed by the "straight-line" method by transfers of capital.

Each property, large or small, offers its own distinct problems of regulation and accounting. Its own management and engineering and accounting officials best know its problems and its needs, and, subject to the approval of the Regulatory Commission having jurisdiction, should be able to estimate the requirements of the property as to proper charges to operating expenses for "depreciation" and proper limits to reserve balances in a far more satisfactory manner than if compelled to use an arbitrary formula which not only deprives the management of control of this important matter, but places restriction on the exercise of informed judgment by the Commission in matters of regulation.

The Society's committee considers that the accounting plan adopted for the creation of reserves for retirement or "depreciation" in any particular case, whether "straight-line" or one of the interest methods, is immaterial, so long as it is founded upon sound economic and legal principles and creates adequate but not excessive reserves. It is the committee's opinion that for most utility properties the results of the application of one of the interest methods, preferably the present worth method, most nearly parallel the depreciation requirements.

The best proof of the results that follow by the use of any method is found in the statistics of properties which have used that method for a long period of years. Regulated accounting has been in operation for a quarter of a century and more statistics are becoming available. The committee has used certain of these statistics to give point to the argument which it presents in support of its views.

The Society's committee believes it proper to state here that all of its members are advocates of the regulation of public utilities; that they fully recognize the great amount of good that has resulted from this form of regulation. They believe that this regulation should be for the benefit not only of the patrons of the utility—the consumers—but also for stockholders and security holders as well; to insure fair and reasonable rates which will permit the utility to maintain its plant, adequately compensate its employees, furnish reliable service, permit a fair return to investors, and maintain the integrity of the investment. The management of a Public Service Corporation has three responsibilities—(1) to furnish the best of service to its patrons at the lowest rates consistent with, (2) the payment of good salaries and wages together with old-age retirement

allowances to skilled employees many of whom devote a lifetime to the service, and (3) a return to the stockholders sufficient to permit the Corporation to attract new capital when needed for expansion.

## II.—Definitions

Among the reasons why the general subject which has come to be called "depreciation" is so confusing and difficult of understanding are: (1) the use of the word depreciation to refer to the amount of physical deterioration or loss of service value to be found and deducted in valuations for the establishment of a rate or taxation base; (2) the use of the same word to refer to an accounting reserve created to provide for the retirement of plant through charges to operating expenses; (3) the frequent use of the word, sometimes expressed as "annual depreciation," to describe the amount of money annually charged to operating expenses and credited to the reserve; (4) the frequent use of the word to describe the credit balance in the reserve, sometimes using the term accrued depreciation for the same purpose; (5) the fact that in the voluminous literature of the subject published in the period, 1912 to 1925, and later, many attempts were made to coin terms to express certain distinctions, such as "physical depreciation" and "functional depreciation," which have come into general use and are well understood; and a number of others used to express some particular shade of meaning which the author was trying to express, and which have not come into general use.

In Chapter II of the NARUC report the nature of depreciation is discussed at length and the present-day concepts are outlined in considerable detail. A number of the definitions current in the literature and in the regulations of various public agencies are presented. It is quite apparent that the term "depreciation" is employed in all of the quoted definitions in a qualitative sense, which is correct because the term means literally the state of being of less worth. However, in valuation and accounting, the objective is quantitative and the long custom of employing the term in both a qualitative and quantitative sense has been in part responsible for the misunderstandings that characterize discussion of the subject.

The Society's committee in its meetings and discussions of the subject has found this confusion of terms to be the cause of much misunderstanding. In order to attempt to make clear to readers of this report the exact meaning attached by the Society's committee to the words used, certain definitions, with comments on each one, are given. It believes that there should be an attempt, on the part of leaders of the engineering and accounting professions, and regulatory bodies as represented by the NARUC, to clarify the subject by the adoption of an approved terminology.

*1. Depreciation.*—

"Broadly speaking depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence. Annual depreciation is the loss which takes place in a year."

Comments: The above is accepted as the best and most authoritative short

definition. It is that of Mr. Chief Justice Hughes in the Lindheimer Case (292 U. S. 151–167). It clearly and definitely refers to the engineering concept—depreciation which is deductible in finding a rate base. It is shorter than the definition in the present NARUC Uniform System of Accounts which reads as follows:

> " 'Depreciation,' as applied to depreciable telephone plant, means loss in service value not restored by current maintenance, incurred in connection with the consumption or prospective retirement of telephone plant in the course of service from causes which are known to be in current operation, against which the company is not protected by insurance * * *. Among the causes to be given consideration are wear and tear, decay, action of the elements, * * * changes in demand and requirements of public authorities * * *."

The definitions of the Interstate Commerce Commission (177 I.C.C. 351-422), 1931; the Federal Communications Commission (Uniform System of Accounts, Telephone Co., p. 4), 1934; the Federal Power Commission (Uniform System of Accounts, p. 5); are practically identical with that of the NARUC. All of them use the term "service value" and all of them exclude all property protected by insurance. These omissions mark the major difference between these definitions and that of Chief Justice Hughes. The definitions of the commissions specify utility property. That of the Chief Justice is a good definition of depreciation of any property, regulated or otherwise. The Chief Justice, instead of using the term "service value," uses the expression "loss * * * due to all factors causing ultimate retirement," a much broader definition.

The definitions of depreciation used by the regulatory commissions have been in use for ten years or more and have controlled in matters of utility regulation. The Society's committee sees no particular objection to any of them, but would call attention to the fact that accounting for the adjustment of insured losses can be handled through the retirement reserve, as well as otherwise, and therefore places no emphasis on that omission from its adopted definition, which is clear cut and otherwise all inclusive.

The NARUC committee recommends the abandonment of all of them and after five pages of argument in its report recommends a new definition which would substitute expiration of "service life or utility of property" for "loss not restored by maintenance" in Chief Justice Hughes' definition, or "loss in service value not restored by current maintenance" in others.

*2. Amortization.—*

The liquidation or extinguishment of the investment in a property by means of a series of partial payments is prorated to extend over the period during which the property will exist, or over the period during which it is anticipated that benefit will be realized.

Comments: The NARUC committee on page XIV limits amortization to recouping the "cost of intangibles, such as patents, copyrights, leaseholds, etc." Depreciation and Amortization are not synonymous. The definition of Encyclopedia Britannica is as follows: "Amortization is the liquidation or extinguishment of an asset by means of a series of partial payments prorated to ex-

tend over the period during which the assets will exist." Amortization is in effect a return to the investors of money expended for wasting assets or for physical plant which is reaching the end of its useful life with no prospect of renewal. This is illustrated by the practice of writing off the investment in coal mining property, oil or natural gas property and the physical plant connected therewith as the coal, oil, or gas supply is depleted. Such utilities as pipe lines for natural gas or oil should create reserves for amortization. In past years, railroads in the lumbering districts built hundreds of miles of branch lines, which were torn up after the lumbering operations were finished. Reserves for amortization of the entire investment, every element of cost, would have been proper, but in many if not most cases were not established.

The Society's committee believes that amortization reserves should be entirely separate from reserves for the retirement of units of property on a continuing utility such as a water works or telephone property. In continuing properties the funds represented by the reserves are not returned to the investor. The object of the reserve is to maintain the integrity of the original investment.

The effect of the forced adoption of the straight-line method in the manner recommended by the NARUC committee is to provide for amortization of an unlimited life property rather than to provide a reserve for depreciation or retirement.

*3. Annual Depreciation.*—As included in the recommended definition of depreciation, annual depreciation is the loss that takes place in a year. The Society's committee believes that long custom has established this term to apply to the estimated annual charge to operating expenses which is credited to the reserve.

Comments: In the body of this report there is a full discussion of the difficulties involved in the exact determination of the amount of depreciation occurring in any one year. One of the fundamental differences between the NARUC report and that of the Society's committee is that the NARUC committee determines annual allowances for depreciation and assumes that the accrued depreciation follows, whereas the Society's committee believes that accrued depreciation should be determined and from accrued depreciation an annual allowance can be set up.

*4. Accrued Depreciation* is the total depreciation that has taken place in a unit of property or an entire existing property from the date when it was first placed in service up to the date of inquiry.

Comments: This term, accrued depreciation, is understood to be synonomous with "actual" depreciation as used by the NARUC committee. Retirements that have actually been made are not reflected in accrued depreciation.

*5. Reserve for Depreciation or Retirements* (hereinafter referred to as "The Reserve").—The accounting reserve is established by a series of annual charges to operating expenses and credits to a reserve, against which the cost of units of property, less net salvage, are charged upon retirement from service for any cause. The object of the reserve is to maintain the integrity of the investment in plant "so that the original investment remains as it was in the beginning."

Comments: The only reason for the reserve that has been approved by a long line of United States Supreme Court decisions is to restore to the property

through this accounting device the full cost, less net salvage, of all units of plant, during the lifetime of the units. The accounting profession in the early history of this accounting device used the term "depreciation reserve" and through the years developed the theory that it should cover all theoretical expired service value or service life of all units, so that the credit balance at the end of any year would include all of the theoretical loss of value of all units in service, many of which would not be retired for many years or decades.

*6. The Credit Balance in the Reserve* is the amount of the book figure after all credits have been added and all retirements and other debits deducted.

Comments: The credit balance in the reserve should be large enough not only to cover all loss in service value presently known but also a reasonable amount to cover sudden retirements caused by storms and other unforeseen casualties.

The Society's committee considers it highly improper to call this credit balance "Accrued Depreciation," a term which should be applied only to total actual impairment to service value—a physical fact.

## III.—Principal Conclusions

The Society's committee, as a result of the study of actual statistics of regulated accounting for so-called "depreciation" over a period of thirty years, and of a review of the U. S. Supreme Court opinions bearing on questions of valuation and depreciation from the early years of this century, has reached certain conclusions, some of which are indicated in the text itself.

1. The summary that is included in the NARUC report states, "In summary statements it is obviously not possible to capture all the refinements of meaning or all the particular qualifications set forth in the main text. Therefore these statements are to be taken as condensed expressions of the ideas and principles that are developed more fully in the report." The report should have either chapter summaries or a general summary of conclusions to which the committee is unequivocally committed.

2. Depreciation is factual and quantitative, capable of reasonably accurate determination by an engineering study of plant and records.

3. The NARUC committee report discusses the application of the straight-line method with judgment in the determination of lives of property units, or groups of units, from mortality tables. The facts are that on large proportions of public utility property there are no reliable mortality data and on many properties of considerable magnitude there may be no mortality statistics at all. The Society's committee believes that the application of general data regarding mortality and the adoption of average lives for various units of utility property are to be strongly condemned. Such a use of any method cannot lead to other than inaccurate and inequitable results.

4. The straight-line method advocated by NARUC committee assumes that a unit of service to be rendered today and one to be rendered some day in the future are of equal value, which obviously they are not. The only theoretically correct method of distributing the depreciation of a unit of property over its

service life is on the basis of a method by which the time value of money is recognized.

5. The Society's committee is unable to see how allowances for depreciation expense can be made to the owners unless it is known with reasonable accuracy at all times just how much of the usefulness of the property has been consumed. It is believed that the NARUC committee should reconsider the statements of approved practice which it recommends to bring them in line with the fundamental purpose of depreciation accountancy, the clear implications of a long line of court decisions, and the plain common sense view that the allowance of depreciation expense is only the method of compensating the owners for property consumed and should in no wise be amortization of principal.

6. The cost of plant and equipment worn out or consumed in producing goods or rendering services is part of the cost of goods or service and should be included in the selling price of the products during the lifetime of the various units of the plant and equipment.

7. Reserves should be created and maintained to offset depreciation. This creation of reserves was first approved in the Knoxville Water Works Case (212 U. S. 1), in which the scope of the reserve was limited to "maintaining the integrity of the investment"; restoring to the property through charges to operating expenses the dollars which represented the cost of equipment retired from service. That principle has been recognized and approved in numerous cases since that time.

8. The Society's committee is of the opinion that, in establishing regulations for the accounting of public utility properties in the various utilities, it would be most unwise to attempt to apply any one formula or rule for universal application in accounting for depreciation. There are such wide differences in the factors causing depreciation in the properties of the different utility industries that the use of any accounting method will revert to estimates, and deal with many uncertainties. In any industry the great differences between small and large plants, or between static and rapidly growing ones, call for individual treatment. The present classification of the NARUC leaves latitude for the exercise of judgment by the various commissions and by the management of the corporations and permits the commissions to base their approval on records of past experience, as has been emphasized in a number of U. S. Supreme Court opinions. The change proposed by the NARUC committee substitutes a particular formula in the application of which adequate data are frequently lacking, and which is complicated by the question of salvage at some remote future date, and takes away in very large measure the opportunity for the exercise of sound judgment based on experience of the commission and the management.

9. The use of straight-line depreciation for thirty or forty years by two great utility industries has been discussed in this report at some length and actual data of its results have been shown. Analysis of these data has convinced the Society's committee that the enforced application of the straight-line method to the majority of public utility properties is not in the public interest.

10. The tables and charts in this report show very definitely that the telephone and railroad utilities, using the straight-line method, have built up credit

balances in the reserve that are vastly in excess of their requirements and are approximately double those of other utilities which have operated under the present NARUC accounting classification.

11. The Society's committee believes that, in cases where adequate mortality data are available, it is possible for an engineer, experienced in design, construction and operation of properties similar to those being studied, to combine the use of such data with his examination of property, and, by the application of one of the interest methods, preferably the present worth method, arrive at proper figures for annual allowances, which will result in reserves that are in fairly close relationship to actual requirements. It is also believed that the application of the same process will give reasonably close estimates of accrued depreciation, but that the major increment in such determination must be the exercise of judgment. The use of mortality statistics largely incidental should not be controlling.

12. The "reserves" in general do not have money or securities in them; they reflect money that has been charged to operating expenses and expended for new plant or other corporate uses. None of this has gone to the stockholders direct. Where reserves are excessive over and above needs for retirement and where the excess has been expended for new plant, the value of their equity has been increased. Analysis of these reserves as made in the pages of this report indicates that in the past the reserves have made very substantial contributions to the capital additions.

13. The reserves, to the extent that they exceed the needs for depreciation, reflect dollars that have been advanced by ratepayers on which the corporation has paid no interest, but which have been expended for new plant. The saving of interest to the corporations through the use of the money collected for depreciation, which otherwise would have been financed by issues of stocks and bonds, is in some cases very large.

The Society's committee believes that this condition resulting from the use of the straight-line method calls for some modified plan of reserve accounting that will prevent such occurrences in the future.

14. Equity requires that the part of the credit balance in the reserve, which is in excess of current requirements, should bear interest.

15. The Society's committee is wholly in disagreement with the conclusion of the NARUC committee that the credit balance in the reserve represents "actual" depreciation and should be deducted in a valuation for rate making. This is incorrect except in those very rare instances where the credit balance in the reserve and the actual requirements are in balance and where a depreciated rate base is appropriate. The NARUC recommendation disregards the purpose for which reserve accounting was established and approved by the Courts, namely, to keep the investment intact and to restore to the corporation the cost of all property retired from service from any cause.

16. In the Progress Report the Society's committee stated[1] its conclusions in reference to the proposed retroactive adjustment of existing retirement reserves to bring the current reserve up to the so-called "depreciation reserve" computed on the straight-line basis through charges to surplus or a special section of the income account. Those conclusions are here repeated:

"* * * The Society's committee feels that in view of the fact that practically all utilities have been subject to governmental regulation over a period of two to four decades and the reserves currently carried on the books of the utilities have been accumulated, credited and charged in compliance with the regulation of such bodies, the adjustment of the current reserve upward so as to equal those computed on the straight-line basis by withdrawal from surplus or otherwise is not only generally financially impracticable but is also ethically unjust.

"Present reserves should, and usually do, reflect rates charged and collected from consumers. To arbitrarily increase the reserves at this late date by withdrawal from surplus is to require the utility rather than its consumers to make good the cost of property used up in their service. This procedure is in conflict with the basic principle of depreciation as well as plain justice.

"The reserve requirements should be measured by the actual needs of the utility and not by an arbitrary straight-line determination which in the case of many of the best utilities in the United States would require reserves substantially double those which a long background of experience under regulation has determined as adequate."

17. The Society's committee is of the opinion, as stated in its preliminary report, "that the actual depreciation should control the size of the reserve, rather than that the reserve should determine the actual accrued depreciation."

18. That method of accounting for depreciation and for the establishment of retirement reserves to provide for future retirements is best which most closely harmonizes with the facts found for each individual property.

19. Uniformity of treatment and administrative convenience in dealing with depreciation are desirable when equitable; but neither uniformity nor administrative convenience should control over the facts of a particular situation. The NARUC report under discussion would make incorrect theories, rigid uniformity, and administrative convenience controlling. Actual depreciation existing in the physical property is a fact. It is the kind of fact that can be determined with reasonable accuracy only by study of the property by qualified, trained engineers or operators with engineering training who follow approved methods.

20. The state regulatory commissions in the United States must be credited with contributing greatly to the abolition of serious evils that once beset the utility industry. The decisions of the commissions have generally been sound and wise. The commissions have had freedom to exercise their own judgment, rather than being bound by arbitrary rules and regulations. The Society's committee believes that accounting regulations which have left much to the wisdom and discretion of the commissions in the matter of depreciation is abundantly proven by the results.

21. Generally speaking, the Society committee's views regarding the proper procedure in determining annual depreciation allowances are as follows:

1. The preparation of an engineering determination of accrued depreciation.
2. The determination of an annual depreciation allowance which had it been in effect during the life of the property would have provided for all retirements and left a credit balance in the reserve approximately equal to the actual depreciation.

## IV.—Historical Development of the Conception of Depreciation

*A. Valuation in the Nineteenth Century.—*

Valuations which were made to determine the price to be paid for property, when the issue was the purchase of a water works by a municipality on the expiration of the franchise; a valuation to establish a basis for taxation on the *ad valorem* method, as was the case in the Michigan valuation of railroads, or to determine a rate base as was true in other states, were the pioneer undertakings in what has come to be a large and important field of engineering practice. In all of them the early work was along practically the same general lines.

The engineers secured from all available sources data as to the units which composed the property—the inventory; they found the actual cost if data were available; they estimated what it would cost to build an identical property at the time of the investigation, thus establishing the cost of reproduction at then prevailing prices; from which was deducted accrued depreciation to arrive at a figure designated as "Cost of Reproduction Less Depreciation."

While this report deals with the problem of depreciation, a brief review of the history of valuation, of which the determination of depreciation is an important element, is essential to an understanding of the origin of some of the theories now prevailing.

There are very few references to the subject of depreciation in the literature prior to 1900. A few old reports are to be found in the libraries, none of which gives any details, but all of which indicate that the subject of depreciation in the sense of loss of value was recognized.

One paragraph in the annual report of John T. Clark, State Engineer of New York, for the year 1855, is worth quoting:

> "It is contended by many skillful managers that if the works and rolling stock are kept in thorough repair, they are in as good condition at the end of each year as they were at the end of the preceding year, and therefore that there can be no depreciation. The difficulty lies in determining the fact whether the repairs have been kept up in this thorough manner."

This indicates that in the very early days of the railroad industry the managers advanced exactly the same arguments that were used in hearings before the Interstate Commerce Commission seventy years later.

In the "Nebraska Rate Cases" which reached the U. S. Supreme Court as the famous Smyth vs. Ames (169 U.S. 466, decided March 7, 1898), there was much testimony as to the cost of building the roads at that date, 1894–1895, and the state based its testimony as to value on the cost of duplication theory at the period of the very lowest price levels recorded since the Civil War. In the often quoted "rule" laid down by the Court in Smyth vs. Ames there is no mention of depreciation. After naming various matters for consideration, the Court added a sentence: "We do not say that there may not be other matters to be regarded in estimating the value of the property." Depreciation has developed as the most important of those "other matters."

In 1897 the first paper on the subject of valuation in *Transactions* of the American Society of Civil Engineers, by the late Wynkoop Kiersted, M. Am.

Soc. C. E., entitled "Valuation of Water-Works Property"[2] discussed Measures of Value. Four members discussed the paper. The author and the four who discussed agreed that "cost of duplication less depreciation" was entitled to the greatest weight.

There are brief references to the accounting for loss of value by the establishment of "sinking funds" by the late Emil Kuichling,[3] M. Am. Soc. C. E., and by the late Alex Dow, Hon. M. Am. Soc. C. E., in 1898 in a paper presented to the National Electric Light Association which give recognition to the practice of private industry in accounting and to the need of some provision by the utilities to care for depreciation, but no general use of reserves was made by the railroads or early utilities. The reason for this was undoubtedly the opinion of the Supreme Court in the earliest recorded attempt to create reserves for replacement.

The Kansas Pacific Case (United States vs. Kansas Pacific Railway Co., 99 U.S. 455, decided October, 1878) dealt with the subject of "depreciation" reserves. The Auditor of the railroad undertook to establish a reserve for the replacement of rails, assuming a 20-yr life, and contended that it was proper to reserve from earnings each year an amount representing loss of life chargeable that year.

The Court said:

> "First. 'Depreciation account or expense not charged up.' This is explained to be the amount necessary to put the road in proper repair, but which was not actually expended for the purpose. We are clearly of the opinion that it is not a proper charge. Only such expenditures as are actually made can with any propriety be claimed as a deduction from earnings."

That opinion clearly disapproved the creation of reserves and was the controlling case for the rest of the century. There was at the end of the century no regulation of accounting of any utility. The Interstate Commerce Commission had been created in 1887, but was not given authority over accounting until 1907; and not until after the first decade of this century were other utilities than the carriers brought under regulation.

Valuation of properties for sale, control of security issues, rates and taxation commenced in a small way in the 1890's. The acquisition of water works plants by municipalities on the expiration of franchises accounted for most of it. A few valuations of railroads under the Texas "Stock and Bond Law" were completed before 1900. The Michigan Railroad Appraisal, begun in 1900, was the first state-wide appraisal in the United States. It was the first to develop rules and uniform methods, and to adopt cost of reproduction less depreciation as the measure of a taxation base. Depreciation was considered as a condition below the maximum service value at which property could be maintained and was based on inspection and the personal opinion of the senior engineers, all experienced railroad men.

This work was followed by state-wide valuations in Wisconsin, Minnesota, Washington, and several other states prior to the beginning of the Federal Valuation of Railroads in 1913. In all of the states the problem dealt with

[2] *Transactions*, Am. Soc. C. E., Vol. XXXVIII (December, 1897), p. 115.
[3] "The Financial Management of Water-Works," by E. Kuichling, *ibid.*, p. 1.

depreciation—a condition of plant to be found and deducted from value. None of them dealt with problems of reserves or of accounting for the purpose of recovering the cost of worn out or retired property through charges to operating expenses. The first extensive consideration of that subject was by the Bell Telephone Utility, discussed at length later in this report.

*B. The Period of the State Valuations.*—

The Texas valuations under the Stock and Bond Law recognized the existence of depreciation, but made no estimate or deduction for depreciation, assuming that the property was maintained in normal operating condition and that reasonableness of security issues was the chief problem to be solved.[4] Existing depreciation became a major consideration of the State Railroad Valuations, 1900 to 1912. The Michigan, Wisconsin, and Minnesota valuations were completed or under way when the Interstate Commerce Commission was first given authority to regulate railroad accounting in 1907. Depreciation was determined by the senior engineers by inspection and represented the personal judgment of experienced railroad men as to the extent of falling below a maximum condition of operating efficiency. A handbook issued by the Master Car Builders Association containing rules for determination of depreciation was used as to all cars.

Measured by standards of valuation practice of later decades, these early valuations were primitive. In most of them the total figure of depreciation was a composite of the personal judgment of many engineers. The Washington State Appraisal was unique in that the field determination of depreciation was done away with. The description of the method used in Washington is quoted from the first extensive paper on the subject of railroad valuation published[5] and is as follows:

> "A percentage of depreciation was not placed in the field, but was determined by 'mortality tables,' or by ascertaining the probable years of structure life, then determining from the age of the particular structure under consideration its percentage of depreciation, a method by no means new."

This is what is now called the "straight-line" method, a term not used in this 167-page paper. It is of further interest to note that all that is contained in the paper on the subject of depreciation are the foregoing paragraph, half a dozen scattered sentences, and the following statements:[6]

> "(*d*) *Depreciation.*—Thus far, this discussion has not dealt at length with the subject of depreciation, and it is not considered essential to the purposes of this paper that it be done. The State appraisals have raised a question as to the propriety of using mortality or life tables as compared with personal inspection and the placing of a percentage based on individual judgment. * * * It is certainly desirable to secure the opinion of the man who inspects a bridge, or building, or locomotive, as to its physical condition. It may be desirable to use the check secured by the fact that the age of the building is known and also the average life of structures of its class."

[4] See "Method Used by the Railroad Commission, of Texas, Under the Stock and Bond Law, in Valuing Railroad Properties," by R. A. Thompson, *Transactions*, Am. Soc. C. E., Vol. LII (June, 1904), p. 328.

[5] "The Valuation of Public Service Corporation Property," by Henry Earle Riggs, *Transactions*, Am. Soc. C. E., Vol. LXXII (June, 1911), pp. 1–91.

[6] *Ibid.*, p. 141.

These references to this old paper presented in January, 1911, are quoted to direct attention to the fact that the problems now under discussion are not new problems, but have been recognized for many decades. That paper was discussed by twenty-five prominent engineers and economists and constitutes a symposium on the subject just before the beginning of the Federal Valuation. One more quotation from the discussion is of interest. It has been noted that the Washington State Appraisal used "life tables," that is to say, the straight-line method, in finding "depreciation." That appraisal was made under the direction of Halbert P. Gillette, M. Am. Soc. C. E. In his discussion[7] Mr. Gillette said:

> "Of almost as great moment as the item of development expense is the question of depreciation. The author [referring to the author of the paper under discussion], in common with most engineers, holds that depreciation should be deducted. This is a consequence of regarding a public service plant as if it were a machine bought in a second-hand store. A public service plant is a device which is intended to perform a given service forever. It is true that its parts are subject to wear, and must be renewed from time to time; but the plant as a whole is everlasting, or practically so. Managers of public service corporations, perceiving this fundamental truth, have rarely established sinking funds for the redemption of any considerable part of the plant. In a great railway system the renewal of a freight car is not a proportionately larger item of expense than is the renewal of a tooth in a steam shovel bucket owned by a contractor. This fact, coupled with the permanence of the railway plant as a whole, has led railway owners to make no provision for a return of the money lost in depreciation. Railway ties in a large railway system inevitably reach a condition such that their average age is exactly half the life of the average tie. Shall a sinking fund be provided for ties? If not, where does logic place a line of demarcation? When does an element of the railway plant attain a condition of sufficient importance to warrant 'writing off' some of its value from the capital account? The facts are that railway managers have not 'written off' anything worthy of mention for depreciation, and, in the writer's opinion, they have been perfectly logical. * * * In other words, if depreciation is to be deducted from the cost of reproduction, it must be added to the development expense ascertained from the accounting records; so that, in the final analysis, depreciation should be ignored entirely in any appraisal of a public service corporation where the object is either rate-making or purchase of the corporation by the public. One qualification to this statement is needed, however, and that is that the depreciation shall not have gone far enough to result in an average age of plant less than half the life of the plant—that being the ultimate normal operating condition."

It is of special interest to note from the above quotation that the only man who used the straight-line method for estimating depreciation in a state valuation of railroads opposed its deduction from value.

The foregoing discussion is designed to throw light on the views of the profession prior to the beginning of the Federal Valuation of Railroads.

*C. The Period of the Federal Valuations.*—

A Committee was appointed by the American Society of Civil Engineers in September, 1911, to Formulate Principles and Methods for the Valuation of

[7] *Transactions*, Am. Soc. C. E., Vol. LXXII (June, 1911), pp. 278–279.

Railroad Property and Other Public Utilities. It submitted a Progress Report in January, 1914. The Final Report was completed in the autumn of 1916 and submitted at the Annual Meeting, January 17, 1917,[8] a few weeks before the United States declared war on Germany. As a consequence of war only four members of the Society discussed this very significant report.

The Federal Valuation of Railroads was provided for in 1912; the Board of Valuation Engineers of the Interstate Commerce Commission was created and organized in 1913. The Texas Midland Opinion of the Interstate Commerce Commission (75 I.C.C. pp. 1–48) fully outlined the methods, rules, and theories as used in the primary valuations of this Commission.

This great undertaking was pushed through to the completion of the primary valuation in the middle 1920's; railroad records were made, such as had never existed before; the most elaborate and accurate studies of costs were completed dealing with prices of the period 1910–1914, which are admittedly as accurate as it is possible to make them, and valuations priced as of date June 30, 1914, were completed for all carriers. Depreciation was deducted on the age-life or straight-line method, the percentages of deduction varying on different carriers but for most of them ranging from 18% to 25%, some running as high as 30% or more.

The only case bearing on the Primary Valuation which reached the Supreme Court was the Los Angeles and Salt Lake Case in which the Court held that the valuation was merely the gathering of data for the files of the Commission which might be used some day in litigation, but the collection of which did not injure the carrier.

The "recapture valuations" which commenced in the middle 1920's made use of the Primary Valuation in the St. Louis and O'Fallon Case (279 U.S. 461). There the Court held that the use of 1914 prices in valuations of 1920 to 1922 and later was improper, and that weight must be given to the price levels of the years of valuation. The Bureau then adopted new methods to translate the primary valuations into the terms of values as of later years. The methods used were radically different from those of the primary valuation.

Many recapture valuations were completed, but no case reached the Supreme Court before the repeal of the recapture provision of the Transportation Act in 1933. The Federal Valuation of Railroads practically ended with that repeal, leaving as its most valuable result the record of the Primary Valuation.

*D. Accrued Depreciation Is a Determinable Fact.—*

The vast majority of public utility properties must be considered as having an indefinite, continuing life, a life that will last as long as there will be a demand for the service being rendered; therefore there is no necessity for amortizing the entire investment. Each property is a composite of many separate, and different, kinds of units, each having terminable life of varying length, each unit being made up of a large number of items which must be kept in a maximum condition of service efficiency and which must be replaced when it ceases to function in order that the unit be always in maximum service condition. Repair and re-

[8] *Transactions*, Am. Soc. C. E., Vol. LXXXI (December, 1917), pp. 1311–1620.

placement of items of property is distinctly maintenance, a straight operating expense. The unit of property, when retired, is written off the fixed capital. The claim of the railroads throughout the entire period of valuation was that the maximum or 100% condition of the unit was not "new" as claimed by the Bureau of Valuation, but maximum condition of service efficiency, with its component items in all stages of life from newly installed to imminent replacement. The Bureau of Valuation formula assumed that such items of the track or road units as ties, or of telephone units as poles, were 100% when newly installed and zero when replaced, thus reducing value of these millions of items to around 50%.

A study of the Primary Valuation reports discloses not only very high percentages deducted from value but some interesting discrepancies due to the assumptions of "life" and the failure to draw a clear line between units, such as major sections of "road" or "track," composed of the items of grading, ballast, ties, rails, etc. As a result carriers with a large investment in right of way and grading show very much less depreciation than those in level territory with little grading.

One of the "best" carriers in the country, with approximately only 10% deduction for "depreciation," was found to be the Bingham and Garfield Railroad, an ore-carrying road a few miles long running from the Great Copper Mines in Bingham Canyon, Utah, to the Smelter on Great Salt Lake. This is a mountain road with very heavy cuts and fills which "do not depreciate" and with heavy steel bridges and trestles of long life.

If there is a better example in America of a single purpose road, with a life limited to the life of the mine, and with a real need for creating reserves for amortization of the entire property, it is hard to imagine. Yet less than half of the "depreciation" was found here than for such carriers as its only connection—the Union Pacific, or the New York Central, Norfolk and Western, Chesapeake and Ohio and other heavy duty highly maintained properties.

The assumption that such land as railroad, power, telephone and other rights of way, and grading, do not "depreciate" is hard to sustain. There are several thousand miles of abandoned right of way and grading of railroads that have been torn up. It is hard to imagine anything more worthless. The grade of a railroad is an element or item of the track or road unit, such as a main-line division or branch line, which calls for constant maintenance to overcome erosion, repair washouts, and similar expense.

Accrued depreciation, deductible in a valuation on any property, is only in part evident on inspection. The full amount of it is a matter of estimate based on studies of past maintenance, study of the history of the property, determination of the amounts of money that should be spent to restore the plant to full condition of operating efficiency, and thorough study of the obsolescent units to determine whether there is justification for their imminent retirement. Engineering valuation has been carried on for nearly half a century. Theories and methods have been developed and have been in constant use for all of that time that are at wide variance with those used by the Interstate Commerce Commission's Bureau of Valuation. They have not been based upon the age-

life straight-line estimates. They have been approved by the Courts as later quotations in this report will indicate.

The railroad contention was that accrued or deductible depreciation in any utility property at any time is an existing condition below the maximum service condition which can and should be continually maintained, a physical fact bearing little relation if any to the theoretical elapsed life of the property. Its only relation to any reserve that may be established for the retirement of the units of property or for the amortization of the whole property in an expiring plant is that it should have been provided for, and that the reserve should be enough larger to fully provide for coming retirements.

This form of depreciation has been recognized by the engineering profession, by the owners of utility property, and by the Courts. There is no need here to quote from the many opinions, beginning with that in the Knoxville Water Case (212 U.S. 1). The Society's committee fully recognizes the fact that existing depreciation must be found and deducted in valuation. It believes that it is a matter of fact, not of theory. It cannot be determined with mathematical accuracy by formula, but is subject to determination within reasonably close limits by men experienced in the design, construction, and operation of utility properties. It is basically an engineering determination. It includes not only inspection of the plant, but detailed examination of conditions not evident to the eye. It calls for studies of past and probable future growth, types of plant and equipment, their functional features, and all matters that would affect their probable future usefulness. Such a study establishes a factual basis for appropriate financial and accounting procedure.

The Society's committee contends that accrued depreciation can only be ascertained accurately by this method.

## V.—Historical Development of Accounting Reserve for Retirement or Depreciation

The two subjects, accrued or deductible depreciation in valuation, and accounting reserves for retirement of plant, which are both generally spoken of as "depreciation," are of vital importance in regulation both to the utilities and to the public. They are not well enough understood by engineers and accountants by whom the amounts of money involved must be determined, by the managements of the corporations or officials of the Regulatory Commissions in charge, or by the patrons of the utilities.

Practically all of the discussions of "depreciation" in papers in technical publications or society proceedings or in books by accounting authorities present the academic theories of the authors without using illustrative material drawn from the records of actual experience of the public utilities and railroads, all of which have been under regulation of accounting for twenty-five or thirty years or more. In the following discussion of accounting reserves the Society's committee has chosen its illustrative material from the voluminous public records of data of existing corporations, records actually made during the 30-yr period of regulated accounting.

The subject known as retirement or "depreciation accounting" deals with vast sums of money charged to operating expenses and credited to reserves

which have been miscalled "reserves for accrued depreciation." The only purpose for which these reserves have been approved by the Supreme Court is to reimburse the corporations for the cost of property that has been retired from service or is being consumed in service and will be retired. These reserves were originally called Reserves for Retirement by the National Association of Railroad and Utilities Commissioners—a correct statement of what they really are.

The last data available to the Society's committee drawn from reports of regulatory commissions indicate that in 1942 the credit balances in these corporation reserves were well in excess of eight billion dollars after charging all previous retirements of plant to them. These figures include the credit balances of the major telephone, electric, gas, and railway equipment reserves. This vast sum of money is stated at this point in the discussion in order to emphasize the fact that the subject is of major financial importance not only to the corporations but to the patrons who furnish the money through payment of rates for service and to the commissions charged with regulation. The reserves should be ample for the purpose of "maintaining the integrity of the investment," a term frequently used in opinions of the U. S. Supreme Court, but as has also been made clear from those opinions, they must not be excessive.

The two subjects which are called "depreciation" have been referred to. There must be a clear distinction in any discussion of the subject between (1) accrued depreciation, loss of service value, which must be found and deducted in valuation for the determination of a rate base, a fact which can be determined with reasonable accuracy by inspection of physical plant and study of maintenance records; and (2) the accounting provision for a return to the company of the cost of property retired and for maintaining a reasonable reserve balance to provide for future retirements of property on which depreciation is now operative, and for casualties of flood, storm, or other disaster. The word "depreciation" has been used to describe this reserve and also to describe the annual charge to operating expense, thus using the one word to describe three different things, two of them having little real connection with true depreciation, which is loss of service value.

*Origin of Reserve Accounting.—*

For more than a hundred years there has been recognition of the sound principle that the owners of any manufacturing, mercantile, or utility business selling goods or services to the public are entitled to charge, as an essential part of the cost of production, all of the legitimate costs of management, all operating expenses including the cost of maintenance and repairs to the plant, and the original cost of all major units of plant and equipment worn out in service and retired. This latter item can best be spread with reasonable uniformity during the useful life of major units by means of yearly accruals to a reserve.

The earliest references to the methods adopted for spreading the cost of plant and equipment over its service life found by the Society's committee dealt with "sinking fund" and "annuities" for marine boilers and hulls in the period, 1810 to 1815.

Between 1845 and 1855 there was considerable discussion of the subject of replacement of railway plant and equipment in England, Holland, and United

States. In 1850 a volume entitled "Railway Economy" by Dionysius Lardner, published by Harper and Brothers in New York, contained the statement:

> "Estimating as nearly as possible the major limit of the cost of relaying, at the end of twenty years, the amount of an annuity (to use a term well understood) must be calculated which at the termination of twenty years would be equal to the cost of laying the rails; and an equivalent sum must be taken annually from revenue and invested at compound interest to meet the exigency."

This volume contains the earliest extensive discussion of the subjects of depreciation and of accounting for the cost of retirement of utility property found by the Society's committee.

*Recognition by the Courts of the Necessity for Retirement Reserves.—*

Not until 1909 did the Courts recognize the character of depreciation and provide, not only for the deduction of accrued depreciation, but also for making provision for maintenance of the integrity of the investment. In Knoxville vs. Knoxville Water Company (212 U.S. 1), Mr. Justice Moody said, on January 4, 1909:

> "Before coming to the question of profit at all, the Company is entitled to earn a sufficient sum annually to provide, not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. The Company is not bound to see its property gradually waste, without making provision out of earnings for its replacement. It is entitled to see that from earnings the value of the property invested is kept unimpaired, so that at the end of any given term of years the original investment remains as it was in the beginning. It is not only the right of the Company to make such a provision, but it is its duty to its bond and stockholders, and, in case of a public service corporation at least its plain duty to the public."

In the following month, February, 1909 (in R. R. Com. of Louisiana vs. Cumberland Tel. and Tel. Co., 212 U.S. 414), the Supreme Court said:

> "That it was right to raise more money for depreciation than was actually disbursed for the particular year there can be no doubt, for a reserve is necessary in any business of the kind, and so it might accumulate, but to raise more than enough money for the purpose and place the balance to the credit of capital upon which to pay dividends cannot be proper treatment."

These two cases are quoted because the opinions were handed down before the public utilities came under general regulation and only two years after the first classification of Railroad Accounts was issued by the Interstate Commerce Commission. Thus, right at the beginning of regulated accounting in the United States, there was clear recognition by the U. S. Supreme Court of the propriety of the creation of reserves to maintain the integrity of the investment by restoration to the corporation of the cost of all units of property worn out and equally clear warning against the accumulation of excessive reserves.

On May 21, 1934, Columbus Gas and Fuel Company vs. Public Utility Commission (292 U.S. 398), the opinion was handed down by Mr. Justice Cardozo. This case, and a companion case (Dayton Power & Light Co. vs. P.

U. Commission 292 U.S. 290), decided on April 30, 1934, involved large allowances for depletion in natural gas fields. The Court said:

> "To withhold from a public utility the privilege of including a depletion allowance among its operating expenses, while confining it to a return of 6½% upon the value of its wasting assets, is to take its property away without due process of law, at least where the waste is inevitable and rapid. * * * Plainly the State must either surrender the power to limit the return or else to concede to the business a compensating privilege to keep its capital intact. * * * It is idle to argue that a company using up its capital in the operations of the year will have received the same return as one that at the end of the years has its capital intact and interest besides."

These cases, and three telephone cases, later quoted in this discussion (the New York Telephone Case, 271 U.S. 23, April, 1926; Smith vs. Illinois Bell Telephone Co., 282 U.S. 133, December, 1930, and Lindheimer vs. Illinois Bell Telephone Co., 292 U.S. 151, April, 1934), are among the many cases which sustain the proposition that the cost of retiring units of property no longer useful in the manufacture of goods or the rendering of public service is part of the cost of service and is properly chargeable to operating expenses and thus paid for by the consumer of the goods or services. The creation of reserves for this purpose and the attempt to spread the cost over the life of the units of property with some reasonable degree of uniformity is fully sustained by a long line of U. S. Supreme Court opinions.

*Accounting for Retirement.—*

As already stated, the creation of reserves to collect the cost of retirement of units of plant during the life of these units has been in use for over a century. It obviously had its beginning in the accounting of manufacturing, mercantile and marine property, and other privately owned property.

Apparently during the nineteenth century some form of interest method was in quite general if not universal use. The early references which the Society's committee has found all speak of "sinking funds" or "annuities." In 1897 Mr. Kuichling[3] said "As the gradual formation of a fund by a series of annual payments is generally understood only in a vague manner outside of financial circles, a few words on the subject may be permitted." In that same decade the published papers of Mr. Dow make it evident that the electric power industry, then in its infancy, recognized the need of reserves but had no definite plans for their creation. Mr. Dow was an advocate of reserves as early as 1895.

## VI.—The Straight-Line Method

The straight-line method was defined, or described, by the late Joseph B. Eastman, on November 2, 1926 (118 I.C.C. 353), as follows:

> "First of all the total depreciation expense is determined by estimating the service value of the property in question. This requires not only a ledger value corresponding as nearly as may be to original cost, but also an estimate of net salvage value at the time of prospective retirement and a similar estimate of service life. Finally the annual depreciation charge is determined by dividing the estimated service value by the estimated service life. The purpose of this method of determining depreciation charges is to spread the entire expense evenly over the property in question."

The accounting profession recognizes the straight-line method as a simple method of spreading the cost of a unit of property over the estimated life of that unit by collecting each year an equal part of the estimated cost less estimated salvage without the use of any interest accrual. In the case of a machine shop, printing office, or other privately owned plant, where the cost of all machines is known, where the sole object is to charge enough for the product to cover the cost of wearing out of the machines which make it, and where "conservatism" (that is, possibly excessive charges to depreciation) is not legally forbidden, the plan undoubtedly has great merit. It is evident that small isolated municipally owned utilities can make good use of reserves collected by this method and undoubtedly there are some special instances of privately owned utility properties which could use the straight-line method to advantage and without serious grounds for criticism.

The "Comments of the Committee of the American Institute of Accountants on the NARUC Committee Report" said of the straight-line method:

> "This Institute has for many years been engaged in a campaign in which it has sought to stress equally strongly the indispensability of accounts and the inescapable limitations on their significance. Straight-line depreciation is a crude process of allocation based on working hypotheses that must be constantly reviewed, and on incomplete and in some cases irrelevant statistics which must be constantly supplemented. To apply to such raw material highly refined mathematical processes would be to create an even more delusive suggestion of certitude than that to which Mr. Justice Jackson referred."

The reference to Mr. Justice Jackson in the above quotation relates to his dissenting opinion in the Hope Natural Gas Case which is in part as follows:

> "To make a fetish of mere accounting is to shield from examination the deeper causes, forces, movement and conditions which should govern rates. Even as a recording of current transactions, bookkeeping is hardly an exact science. As a representation of the condition and trend of a business, it uses symbols of certainty to express values that actually are in constant flux. * * * However our quest for certitude is so ardent that we pay an irrational reverence to a technique which uses symbols of certainty, even though experience again and again warns that they are delusive."

Mr. Justice Brandeis, who had an exceptionally clear understanding of accounting problems, in his dissenting opinion in United Railways and Electric Company vs. West (280 U.S. 234, 276–277, January, 1930), discussed the subject of accounting reserves in part as follows:

> "For an annual depreciation charge is not a measure of the actual consumption of plant during the year. No such measure has yet been invented. There is no regularity in the development of depreciation. It does not proceed in accordance with any mathematical law. There is nothing in business experience, or in the training of experts, which enables man to say to what extent service life will be impaired by the operations of a single year, or of a series of years less than the service life.[13]"

Footnote 13 of the opinion reads as follows:

> "Depreciation of physical units used in connection with public utilities, or, indeed, with any other industries, does not proceed in accordance with

any mathematical law. * * * There is no regularity in the development of the increasing need for repairs; there is no regularity in the progress of depreciation; but, in order to devise a reasonable plan for laying aside allowances from year to year to make good the depreciation as it accrues, and to provide for the accumulation of a sum equivalent to the cost less salvage of a unit by the time it is retired, some theory of depreciation progress must be assumed on which such allowances may be based. 81 Am. Soc. Civil Eng. Transactions (1917) 1311–1462–3."

This quotation by Mr. Justice Brandeis is from the report of the Society's Committee on Valuation hereinbefore mentioned,[8] of which the late Frederic P. Stearns, Past-President, Am. Soc. C. E., was Chairman. This long dissenting opinion of Mr. Justice Brandeis, supplemented by that of Mr. Justice Stone who agreed on the subject of depreciation, constitutes one of the most complete legal discussions on the subject extant. Only a few more sentences of this opinion of Justice Brandeis need be quoted:

"The main purpose of this charge (to operating expenses for 'depreciation') is that irrespective of the rate of depreciation there shall be produced, through annual contributions, by the end of the service life of the depreciable plant, an amount equal to the total net expense of its retirement. To that end it is necessary only that some reasonable plan of distribution be adopted. Since it is impossible to ascertain what percentage of the service life is consumed in any year it is either assumed that depreciation proceeds at some average rate * * * or the annual charge is fixed without any regard to the rate of depreciation. * * * It is a bookkeeping device introduced in the exercise of practical judgment to serve three purposes. It preserves the integrity of the investment. * * * It serves to distribute equitably throughout the several years of service life the only expense of plant retirement which is capable of reasonable ascertainment—the known cost less the estimated salvage value. And it enables those interested * * * to ascertain as nearly as possible the actual financial results of the years operations."

Every time $1,000 is charged to operating expenses for "depreciation" and credited to the "reserve," $1,000 is held back from net revenue, from the stockholders, and left as cash for the management to spend for new plant or for other legitimate corporate uses. There are a number of corporations which invest part of the money in securities until needed for construction but the general practice on expanding properties is to invest it in new plant. All retirements, less salvage, are charged to the reserve. The resulting credit balance is a bookkeeping entry representing all prior charges to operating expense, less all retirement. It is usually not maintained as a cash reserve. It is never available to the investor except in case of partial or complete liquidation of the property.

The expenditures for new plant to the extent of past retirements assist in fulfilling the function of "maintaining the integrity of the investment." It is new property replacing retired units of property and to that extent the method should not be a matter of controversy. That, however, does not relate to the credit balance in the reserve as upon the retirement of plant the resulting charge reduces the credit balance. Property previously retired is not represented in the credit balance of the reserve which is available for unmatured depreciation.

Some form of a reserve is essential to provide for future retirements. There

can be no legitimate objection to a reserve, or surplus, of reasonable size to provide for future retirements, some of which can be foreseen with reasonable certainty a few years in advance, and some of which are the result of flood, sleet and ice storms, lightning or other disasters which frequently take a heavy toll of railroad, power, telegraph and telephone property. Such reserves have been approved again and again by the U. S. Supreme Court, but the Courts have been interested in the size of the reserve rather than the method of accrual. The Courts' warnings against excessive reserves, the obviously excessive nature of "straight-line" accumulations, and the fact that in many litigated cases accrued depreciation has been found to be much less than the straight-line reserve, all point to much lower provisions for retirement than have been used in the past by many corporations. The U. S. Supreme Court's decisions in a number of cases refer to experience as the proper guide to the establishment of reserves.

It is very clear, in the light of many U. S. Supreme Court decisions, that there is no legal justification for making provision for anything more than reserves sufficient to reimburse the corporation for the cost of all plant retired for any reason and for maintaining a balance sufficient to provide for depreciation in progress but not yet reflected in retirements. Any theory or any method of accounting that builds up reserves in excess of a reasonable balance or surplus for that purpose is almost certain to lead to controversy and litigation.

It is also true that, at the present time, the law is clear that credit balances that have been accumulated in the past under regulated accounting belong to the company. That fact, however, does not justify continuing charges to operating expenses at rates that are in excess of the actual provable requirements.

In the New York Telephone Case (271 U.S. 23–32), in 1926, the Court said:

> "It may be assumed * * * that in prior years the company charged excessive amounts to depreciation expense and so created in the reserve account balances greater than required adequately to maintain the property. * * * The revenue paid by the customers for service belongs to the company. The amount, if any, remaining after paying taxes and operating expenses, including the expense of depreciation, is the company's compensation for the use of its property * * * customers pay for service, not for the property used to render it. * * * Property paid for out of moneys received for service belongs to the Company, just as does that purchased out of proceeds of its bonds and stock. * * *"

This case was approvingly quoted by Chief Justice Hughes in Smith vs. Illinois Bell Telephone Company (282 U.S. 133–158) in December, 1930:

> "While it has been held by this court that property paid for out of moneys received for past services belongs to the company, and that the property represented by the credit balance in the reserve for depreciation cannot be used for imposition of a confiscatory rate (citing 271 U.S. 23) it is evident that past experience is an indication of the company's requirement for the future. The recognition of the Ownership of the property represented in the reserve does not make it necessary to allow similar accumulations to go on if they are excessive."

It is significant that the court suggests a change in the annual allowance as contrasted to the NARUC Retroactive Adjustment of the reserve.

The inevitable conclusion to be reached from a study of the opinions of the

U. S. Supreme Court is that any method that is shown by experience to provide a sufficient reserve is legal. "Some average rate * * * or the annual charge is fixed without any regard to the rate of depreciation" would satisfy Mr. Justice Brandeis' requirement. All of the opinions limit the amount of money to be raised to the amount of the retirements, to "maintaining the integrity of the investment." The capital of the corporation must be furnished by stock and bondholders—not by ratepayers. Capital consumed by the wearing out of plant in furnishing service is to be restored to the property by the consumers through charges to operating expenses. It must again be emphasized that this is in no sense a return of money to the investors whose money built the plant. It is money that is to remain in the plant to restore the loss in service value caused by depreciation.

The essential difference between the thoughts expressed by the two committees is that the report of the NARUC committee says in substance that the reserve, when properly computed, is the best measure of accrued depreciation, whereas the Society's committee states that the accrued depreciation is factual in nature and that the amount of such accrued depreciation should be included in the amount of the reserve. No member of the Society's committee would take exception to the use of the reserve when properly computed as a measure of accrued depreciation, but the NARUC committee goes further and recommends (Conclusion 26) "The straight-line method is generally recommended for public utility accounting and financial purposes and also for the computation of both depreciation expenses [annual allowances] and accrued depreciation for purposes of rate making." Such a method, in the opinion of the Society's committee, is not a proper one, if the reserve is to be used as a measure of accrued depreciation. Objection is taken to the use of a reserve as a measure of accrued depreciation, unless the reserve is in reasonable conformity with the actual depreciation.

Accrued depreciation, which manifestly involves many variables, should be determined on its own merits, and the reserve accumulated to cover it, and adjusted to it, by means of changes in the annual allowances. It would seem that such a plan would be the only means available for checking the propriety of the annual allowances. The basic thing sought is the determination of actual depreciation, and the provision of a reserve that is at least equal in amount to it, which reserve is accumulated through annual allowances for depreciation. To fix the accrued depreciation by the reserve set up by allowances, determined by any method, is fixing an item factual in nature, and possible of determination within reasonable limits, by an item that should be subject to frequent changes as conditions change. The object of the reserve is to provide for the actual depreciation and not to have the actual depreciation (a factual matter which cannot be changed) equal to the reserve. It is axiomatic that accrued depreciation which automatically integrates all the time variables can be more accurately estimated than the annual allowance. It therefore should be the starting point.

The NARUC committee recommends that the actual depreciation should be determined by application of the straight-line method, which entails the assumption of probable average useful lives for the different units or groups of

units involved, and that the loss in service value would occur in equal annual amounts. The facts are that most adherents to the straight-line theory, of necessity, use tables or probable useful lives prepared by different authorities, mostly general in nature, and having no connection with the property being studied. The term, "of necessity," was used above as very few properties have mortality statistics that can be considered more than roughly indicative, and usually those that have such data do not have them for anything other than mass property. Most mortality statistics relating to mass property are derived from dollar ages, the only ages readily available, rather than property ages. With widely fluctuating prices over the years, the difference may be very substantial. It is probable that no public utility property can ever have mortality statistics on any large percentage of its plant account. Such statistics are of little conclusive value unless they are in the nature of mass data. No property will have mass data on retirements of such items as buildings, boilers, turbines, water purification plants, pumping equipment, and the like. It is probable that the average utility plant, with the possible exception of water works and gas properties having a majority of their investment in pipe lines, would never be able to have conclusive mass data on more than a small percentage of its property.

Even when mortality statistics are available, they cover, generally speaking, units that are not the same as those upon which the amount of accrued depreciation is being determined. Also they cover units retired during a different period and under different conditions. They also cover premature retirements due to casualty, such as storms, fire, automobile collisions and the like. It would seem illogical to say, for instance, that a new pole line had accrued depreciation existing at the start of its use, due to the fact that it might be wrecked by storm during the first year of its life. Saying it another way, the service value of the pole line at any one time would be less because some day the line might be wrecked by storm. That is exactly what takes place when age-life methods are used with useful life figures based upon mortality statistics that include such casualties.

There are many reasons why mortality statistics covering units of property in public utility systems should be used with care, as a basis of determining useful lives for use in age-life calculations. The main reason is that such statistics do not now, and probably never will, show any consistency of "dispersal pattern." The NARUC report likens mortality statistics obtained from utility retirement records to vital statistics upon which life insurance premiums are based, and states that such mortality statistics can be used as a safe means of useful lives for utility property. Vital statistics used by life insurance companies do show a fairly constant "dispersal pattern," but even those statistics are not infallible. This fallibility was recognized by Mr. Justice Brandeis in his dissenting opinion in Pacific Gas and Electric Company vs. San Francisco and in which Justice Holmes concurred, as follows:

> "A depreciation charge resembles a life insurance premium. The depreciation reserve, to which it is credited, supplies insurance for the plant against its inevitable decadence, as the life insurance reserve supplies the fund to meet the agreed value of the lost human life. To determine what

the amount of the annual life insurance premium should be is a much simpler task than to determine the proper depreciation charge. * * * Moreover, for human lives, mortality tables have been constructed which embody the results of large experience and long study. By their use the required premium may be fixed with an approximation to accuracy. But, despite the relative simplicity of the problem, it was found that the variables leave so wide a margin for error that premiums fixed in accordance with mortality tables work serious injustice either to the insurer or to the insured. Although the purpose was to charge only the appropriate premium, the transaction resulted sometimes in bankruptcy of the insurer; sometimes in his securing profits which seemed extortionate; and rarely, in his receiving only the intended fair compensation for the service rendered. Because every attempt to approximate more nearly the amount of required premium proved futile, justice was sought by another route. Ultimately, strictly mutual insurance was adopted. * * *"

Even though it were possible to forecast accurately the life of a property or unit, the parallel between insurance and straight-line depreciation of utility property ends there, for insurance premiums are based on interest accumulating annual payments while straight-line depreciation neglects the interest factor.

It is apparent that very little conclusive mortality data exist so far as utility property is concerned, and that many of the available data are not properly usable as the basis of useful lives for a determination of accrued depreciation, although they may be valuable in its determination. This means that the selection of useful lives for use in age-life methods must be largely a matter of judgment.

One great difficulty in making the straight-line method seem practical to those experienced in the operation of utility property is the fact that, when it is applied to most properties, it results in many absurdities. There will usually be a number of items, actually in use, that should not be operating if the application of the straight-line method using customary useful life figures is accepted as gospel. The usual useful life placed on power plant buildings is 50 years. To illustrate by an actual case, a building from which, at the time it was 50 years old, the old equipment was removed, a new turbine installed, the building given minor repairs, from every known standpoint could be said to have as much value as a new building starting on a new life cycle. If the usual application of the straight-line method had been applied, this building would have been entirely written off about the time it received its new lease on life. The result of such accounting would have been that the ratepayers contributed the entire cost of building through annual allowances for depreciation, during what appears to be perhaps half its useful life. There are other examples, such as turbines that are topped, meters and transformers that are rebuilt, etc., which make the accurate estimating of useful lives very difficult.

When applied to most utility properties, the application of the straight-line method of determining accrued depreciation produces reserves that are so greatly in excess of the actual depreciation that the answer is manifestly wrong. The fault probably lies partly in the improper determination of useful lives in the particular property being studied, and is principally due to the fact that in actual practice depreciation does not take place in a straight line. The use of any age-life method of computing accrued depreciation, whether straight line,

sinking fund, or present worth, is subject to the same inaccuracies so far as selection of useful lives is concerned. It is believed, however, that the interest methods more closely parallel the accrual of actual depreciation.

Perhaps the strongest argument against the use of any mathematically theoretical method of computing accrued depreciation is the fact that actual retirements in seasoned utility properties indicate the complete lack of a sound basis for them. The straight-line method is used quite extensively in industrial properties, and properly so in most cases, as the useful lives of the units making up a large part of the investment are short and amortization as well as depreciation is involved. In the case of utility properties, the reverse is true, as a majority of the units involved have relatively long lives, the property as a whole is continuing, amortization is not a factor in most cases, and equity to the consumers requires that the depreciation reserve be accumulated only as fast as the property is consumed in their service.

It is believed that the statement in the NARUC report to the effect that the use of the straight-line method of figuring both annual allowances for depreciation and accrued depreciation is in the ascendancy expresses a wish rather than a fact. It is true that certain governmental regulatory agencies favor such a plan, but it is certain that, in the field of the public utilities, there is no such ascendancy for determining accrued depreciation. In fact, as depreciation accounting becomes better understood, quite the reverse is true.

As stated previously, the Society's committee takes the stand that accrued depreciation is factual in nature and that accounting should fit the fact rather than let some uniform accounting rule determine what the facts are. This committee further states that, while accrued depreciation is a fact, its determination cannot be made by any mathematical formula or calculation, but must be based upon the consideration of all conditions existing, by someone thoroughly familiar with the design, construction, and operation of the property involved.

## VII.—The Straight-Line Method was Developed by the Bell Telephone Companies

The earliest extended discussion in the present century of accounting reserves is to be found in the Annual Reports of the American Telephone and Telegraph Company. These were all signed by Theodore N. Vail, President.

Beginning with the report of 1907, under the captions, Maintenance and Reconstruction, it is pointed out that from 1903 to 1907 about $150,000,000 was spent on maintenance and reconstruction on plant which in 1907 had a value a little under $500,000,000.

In 1908 the report shows an expenditure of about $40,000,000 for these purposes, discusses "Permanency of Plant," and, under the caption "Depreciation," reports that studies were made during the year "to determine the actual rate of depreciation that should be provided for."

Beginning on page 5 of the report of 1910, under the caption "Depreciation," there is a long discussion on the subject of reserves from which the following quotations are taken:

"The question of depreciation has been considered very critically and analytically during the past year, by commissions and other bodies, in connection with studies on the rate question. While a depreciation reserve was generally favored, there seemed to be a disposition to apply experience and theories, gleaned from other lines of business, to the telephone business.

* * *

"The present policy of the Bell System is to provide against every probable contingency and to base the amount and extent of such provision on past experience—not on future expectations.

* * *

"All of this has been made possible through the unremitting study and research of the staff of the Engineering and Experimental Departments of the Company, who by close attention, observation and study, anticipate and provide for all such contingencies and conditions as can possibly be anticipated or provided for in advance."

The next four or five paragraphs discuss the rapid development of the industry. Mr. Vail cited the need for improved construction in advance of hand-to-mouth needs of the day and referred to work then in progress which was estimated to cost $180,000,000, which, if it were to be built piecemeal, the portion then urgently needed would cost $112,000,000 at the time, but the whole job which was then under construction would total $250,000,000. Thus immediate construction of the whole job resulted in an estimated saving of $70,000,000. The subject is then closed with the following paragraph:

"The capital for this advance construction must be provided by and at the cost of the present, as was the advance construction of the past provided by and at the cost of the past. To the extent that advance construction reduces the cost of necessary plant and anticipates reconstruction and replacement, to that extent the capital charge to be borne by present and future is reduced and to that extent it immediately puts the depreciation reserve to its intended use. The criticism that any excess of reserve is at the cost of the present for the benefit of the future is true, but only to the extent that it may be found eventually in excess of actual retirements. In any case it would be no more than might rightly be considered on insurance against obsolescence which cannot be foreseen."

In the 1911 Annual Report of the American Telephone and Telegraph Company, on pages 5 to 11, Mr. Vail went to the extreme in his argument for reserves created by charges to operating expenses. Space is available in this discussion for only one quotation of the many which the Society committee would like to use in presenting the argument made at that time for straight-line reserves.

"There are however, in the conduct of business many conditions and possibilities which can not be met out of current net revenue and should not be met out of capital, but which if not provided for in some way would put all industrial companies on a speculative basis.

"There is that obsolescence which comes from revolutionary improvements necessitating wholesale replacement of obsolete apparatus or plant, such as the replacement of overhead systems by underground systems, or such as took place when the present method of operating was introduced. There are those fluctuations in net revenue caused by business depression which cannot be overcome by immediate reduction of fixed charges, overhead expense or operating costs. There is that constant tendency to increase in

wages and cost of material, that tendency to increase in operating expenses and capital charges caused by constant demand for increased efficiency of service, that demand for extensions, productive and unproductive, and that call for improvements in plant, equipment and apparatus. There is that increase in costs of operating in greater ratio than the increase in business, peculiar to some branches of the telephone service.

"These and many other possibilities always confronting industrial and public service undertakings must be provided for. They are not the subject of capital expenditures, and can only be provided for by an accumulating surplus and reserves invested in productive plant or securities. If these are not provided for, trouble if not disaster or destruction is inevitable."

In the report for the year 1913, on page 10, is the following paragraph:

"All of the present surplus and reserves aggregating $174,000,000 is invested in tangible and productive property the revenue from which enables the companies to maintain their efficiency, without paying capital charges on this amount."

The previously quoted paragraph from the Louisiana Telephone Case in 1909 seems to indicate that the Telephone Companies had reserves as early as that which were large enough to create the impression that this form of accounting (straight line) was creating excessive accumulations of money which were being invested in new plant.

The studies made by the Bell Telephone officials in the earlier years, 1905 to 1909, resulted in the development of the Accounting Classification of the Bell Telephone Company about 1909 or 1910. The classifications of the New York Public Service Commission and of the Interstate Commerce Commission issued in 1911 or 1912 were practically verbatim copies of the Telephone Company Classification with some slight rearrangement of paragraphs.

The Company has what are probably the most complete records of the life history of property retired over a long period to be found in any utility. Life estimates are based on these studies. Equally fine are the studies of retirements, salvage, cost of removal, etc. The many reports to state and federal commissions, exhibits in the many litigated cases of record, and other published statistics of the telephone utility, make available a great volume of statistical data. There are probably no more complete and accurate records extant than those of the Bell Telephone Companies.

## VIII.—The Experience of the Telephone Utility with Accounting by the Straight-Line Method

In view of the great mass of statistical information available to the state and federal regulatory commissions and other governmental agencies it is a matter of considerable surprise that the NARUC committee in its study, extending over four years, did not investigate the actual results of the accounting methods in use by the telephone, railroad and electric utilities instead of setting up hypothetical computations.

The data drawn from various unimpeachable sources as to the actual experience over a long period of years furnish the best proof of the results of any particular method of accounting. Whether the results are adequate for the purpose or excessive, whether the hidden benefits, such as saving of interest to a

corporation by the use of money represented by the credit balance in the reserve is substantial or not, can only be determined by an analysis of the record.

The Society's committee has made a partial study of the Bell telephone records available to its members. In order to obviate the use of many tables, and avoid great masses of dollar figures running into hundreds of millions or billions of dollars; and to reduce the figures to terms that will permit comparison of the results secured by the use of different accounting methods by different utilities, the tables showing charges to operating expenses, retirements, and growth of credit balances are generally stated in terms of percentage of fixed capital or depreciable plant. To indicate the magnitude of the amounts of money involved, the summaries of the conclusions of the committee are stated in dollar figures.

The telephone utility had its first beginnings in the late 1870's or early 1880's. During the last decade of the past century and the first decade of this one was the period of promotion when scores or hundreds of local telephone companies were organized in many cities and small areas; when such long-distance lines as existed were almost wholly in the hands of the Bell System and were in the process of expansion and development of equipment, and when many of the smaller companies and local systems were being absorbed by the Bell and other large companies.

During the next three decades the process of consolidation of companies, retirement of duplicate plant and of old and obsolete properties, and of invention and development went on. The annual reports of the Federal Communications Commission show that in 1930 there were 136 reporting corporations, whereas in 1938 there were only 73. In 1938 the Commission reported for these 73 companies the values listed as Item 1, Table 1. For the same year, Item 2

TABLE 1.—DEPRECIATION RESERVE DATA FOR 1938; REPORTS OF THE FEDERAL COMMUNICATIONS COMMISSION

| No. | Description | Total investment in plant (dollars) | DEPRECIATION RESERVE | |
|---|---|---|---|---|
| | | | Dollars | Percentage of total |
| 1 | 73 telephone companies | 4,783,082,000 | 1,316,368,000 | 27.52 |
| 2 | Bell Telephone System | 4,489,078,000 | 1,249,563,000 | 27.84 |
| 3 | All other properties | 294,004,000 | 66,805,000 | 22.72 |

gives the statistics of the Bell System, which would indicate, for all other properties, the values given as Item 3, Table 1. The total credit balance in Item 2 is 27.84% of the total plant investment, or 30.1% of the depreciable plant. Obviously either the small properties have been unable to build up the reserves as rapidly as the Bell System or they began reserve accounting at a later date.

The statistics of the one system which has 93.85% of the total telephone investment in the country furnish an excellent illustration of the operation of the straight-line method.

From the early annual reports of the American Telephone and Telegraph Corporation we find that in 1907 the total investment in plant was a little under

$500,000,000. On December 31, 1913, this had increased to $820,885,000 of which 90% was classified as depreciable. The investment in plant steadily increased until it reached $4,183,236,000 in 1931 and for four years was practically stationary or slightly lower. In 1935 it was $4,187,790,000 since which it steadily increased to $5,387,790,000 on December 31, 1943.

During the period from December 31, 1913, to December 31, 1943, the credit balance in the depreciation reserve increased from $107,081,000 to $1,713,175,000. Thus it will be noted that while plant investment increased 556⅓% during the 30-yr period the credit balance in the reserve increased 1500%.

A careful study of the growth of the credit balance reveals the fact that, while its growth, in terms of percentage of depreciable plant, was negligible between 1917 when it was 20.55% of plant value, and 1933, when it stood at 22.63%, the lowest intermediate figure being 20.39%, the highest 25.39%, in terms of percentage of depreciable property the credit balance has increased, in dollars, every year since 1933 and at the end of 1943 stood at 34.34%, or more than one third of depreciable plant.

When the increase of the credit balance in dollars is considered, however, it is to be noted that there has been an increase every year in the terms of tens to hundreds of millions of dollars.

Col. 1, Table 2, shows the growth, in dollars, of the depreciation reserve as above analyzed, and Col. 2 shows the dollars of increase in each year. The total increase in the credit balance was $1,605,994,000 in the 30-yr period of which $914,923,000 came in the 11-yr period from December 31, 1932, to December 31, 1943, during which the investment in depreciable plant increased $1,138,467,000. Thus the money reflected by the increased credit balance in the reserve would have been sufficient to pay for more than 80% of the plant growth.

If Table 2 be again referred to it will be noted that from 1913 to 1926 the annual charges to operating expenses for "depreciation" (Col. 4) were in excess of the total retirements (Col. 5) in all years except one, while from 1927 to 1943 total retirements were substantially above the annual "depreciation" charge in all but the years 1935 and 1943. This fact apparently furnishes justification for the substantial percentages charged to operating expenses, but on the other hand is in conflict with the rapid growth of the credit balance. This calls for analysis.

Study of the statistics shows that plant retired over the entire period cost $4,184,461,000 while total charges to operating expenses amounted to $3,833,435,000, or $351,026,000 less than total retirements, whereas as already noted the credit balance increased more than one billion six hundred million dollars. This directs attention to salvage. The straight-line formula calls for an estimate of life of property and for an estimate of salvage. On actual retirement cost of retired property less net salvage is charged to the credit balance. Data for the analysis of the actual salvage of this more than four billion dollars worth of retired property are not available to the Society's committee, nor are they essential. Accounting accuracy has been almost a religion with the telephone staff for a third of a century and the results of that accounting may be accepted as correct. The point which the Society's committee desires to bring

out is the extreme difficulty which will be encountered when an attempt is made to check salvage records kept under the straight-line method, or to order changes in that method, on a great far-flung property with an investment of hundeds of millions of dollars and with an accounting system less complete than that of the Bell Telephone Companies.

TABLE 2.—TELEPHONE RESERVE ACCOUNTING, ALL COMPANIES FORMING THE BELL SYSTEM

(Percentages in Terms of Depreciable Plant)

| Year | Credit balance in depreciation reserve (thousands of dollars) (1) | Annual increase in credit balance (thousands of dollars) (2) | Credit balance in percentage of depreciable plant (3) | Charge to operating expense for depreciation (percentage of depreciable plant) (4) | Total retirements (percentage of depreciable plant) (5) |
|---|---|---|---|---|---|
| 1913 | 107,181 | .... | 14.55 | 5.01 | 5.10 |
| 14 | 122,338 | 15,157 | 15.58 | 5.15 | 5.07 |
| 15 | 142,307 | 19,969 | 17.38 | 5.38 | 4.60 |
| 16 | 168,044 | 25,737 | 19.25 | 5.63 | 4.55 |
| 17 | 201,090 | 33,046 | 20.55 | 5.35 | 4.26 |
| 1918 | 235,395 | 34,305 | 22.32 | 4.96 | 3.70 |
| 19 | 276,304 | 40,909 | 24.52 | 5.64 | 3.80 |
| 20 | 303,076 | 26,772 | 24.91 | 5.37 | 3.59 |
| 21 | 343,331 | 40,255 | 25.06 | 5.14 | 3.60 |
| 22 | 387,148 | 43,817 | 25.39 | 5.23 | 4.00 |
| 1923 | 434,129 | 46,981 | 24.78 | 4.97 | 4.06 |
| 24 | 476,374 | 42,245 | 23.07 | 4.87 | 4.37 |
| 25 | 519,023 | 42,649 | 22.33 | 5.01 | 4.80 |
| 26 | 564,541 | 45,518 | 21.86 | 4.98 | 4.93 |
| 27 | 584,762 | 20,221 | 20.93 | 5.05 | 5.51 |
| 1928 | 634,030 | 49,268 | 20.90 | 5.03 | 5.72 |
| 29 | 681,611 | 47,581 | 20.39 | 4.88 | 6.15 |
| 30 | 720,690 | 39,079 | 19.68 | 4.92 | 6.63 |
| 31 | 768,817 | 48,127 | 20.14 | 4.96 | 6.42 |
| 32 | 798,252 | 29,435 | 20.73 | 4.63 | 6.69 |
| 1933 | 876,373 | 78,121 | 22.63 | 4.34 | 4.96 |
| 34 | 950,628 | 74,255 | 24.46 | 3.87 | 4.22 |
| 35 | 1,042,011 | 91,383 | 26.65 | 4.31 | 4.20 |
| 36 | 1,124,809 | 82,798 | 28.38 | 4.06 | 4.30 |
| 37 | 1,195,269 | 70,460 | 29.43 | 3.98 | 4.66 |
| 1938 | 1,249,563 | 54,294 | 30.10 | 3.79 | 4.50 |
| 39 | 1,300,660 | 51,097 | 30.72 | 3.78 | 4.97 |
| 40 | 1,357,091 | 56,431 | 31.04 | 3.74 | 5.08 |
| 41 | 1,440,224 | 93,133 | 31.12 | 3.74 | 4.73 |
| 42 | 1,556,095 | 115,871 | 31.85 | 3.82 | 4.11 |
| 1943 | 1,713,175 | 157,080 | 34.34 | 3.95 | 2.86 |

Table 3 is derived from a table introduced by the New York Telephone Company and marked for identification in connection with the cross examination of a nationally known authority on accounting at Albany, N. Y., April 15, 1930. It is a summary of gross retirements of that company for the period 1910–1929 inclusive, sales excluded, showing the average percentage that each class of property is of total property retired during that 20-yr period together with the portion of original cost recoverable through salvage for each of these classes of property.

This 20-yr average percentage of net salvage on the retirement of nearly 193 million dollars does not, however, furnish any clue to the average annual net salvage from retirements during the 20-yr period. During these years the

lowest percentage was 24.62%, the highest 59.02%. The average percentages for five years were less than 40% and five above 50%. This discussion of actual salvage experience is introduced to make it perfectly clear that the estimates of the "life" of plant or of "salvage" are not the simple matters that are indicated

TABLE 3.—TOTALS OF PLANT OF VARIOUS KINDS WITH NET SALVAGE THEREFROM RETIRED DURING PERIOD 1910–1929, INCLUSIVE

| Acct. No. | Portion of property | Original cost, sales excluded | Net Salvage; Sales Excluded | |
|---|---|---|---|---|
| | | | Dollars | Percentage of original cost |
| (1) | (2) | (3) | (4) | (5) |
| 207 | Rights of way | 932,166 | 11,433 | 1.2 |
| 212 | Buildings | 6,253,816 | 1,020,156 | 16.3 |
| 221 | Central office (C.O.) telephone equipment | 31,717,472 | 4,416,055 | 13.9 |
| 222 | Other equipment of central office | 3,453,943 | 1,647,833 | 47.7 |
| 231 | Station apparatus | 54,691,545 | 40,384,344 | 73.8 |
| 234 | Private branch exchange (P.B.X.) switchboards | 31,473,557 | 21,313,744 | 67.7 |
| 234 | Booths and special fittings | 3,815,608 | 3,091,958 | 81.0 |
| 241 | Exchange pole lines | 8,935,427 | 1,909,547[a] | 21.4[a] |
| 242 | Exchange aerial cable | 7,858,225 | 1,547,781 | 19.7 |
| 251 | Toll pole lines | 4,390,289 | 176,142[a] | 4.0[a] |
| 252 | Toll aerial cable | 726,134 | 345,036 | 47.5 |
| 253 | Toll aerial wire | 3,073,548 | 1,285,938 | 41.8 |
| 244–14 | Exchange underground (U.G.) conduit main | 302,179 | 8,655[a] | 2.9[a] |
| 244–24 | Exchange underground conduit subsidiary | 123,459 | 4,673 | 3.8 |
| 245–15 | Exchange underground cable main | 9,543,735 | 5,477,389 | 57.4 |
| 245–25 | Exchange underground cable subsidiary | 7,289,682 | 165,782 | 2.3 |
| 246 | Exchange submarine cable | 270,556 | 52,672 | 19.5 |
| 254 | Toll underground conduit | 13,591 | 5,385 | 39.6 |
| 255 | Toll underground cable | 2,992,005 | 1,866,188 | 62.4 |
| 256 | Toll submarine cable | 329,493 | 105,750 | 32.1 |
| 261 | Office furniture and fixtures | 2,186,961 | 610,185 | 27.9 |
| 262 | General shop equipment | 67,734 | 29,492 | 43.5 |
| 263 | General store equipment | 206,897 | 74,400 | 36.0 |
| 264 | General stable and garage equipment | 3,547,095 | 563,462 | 15.9 |
| 265 | General tools and implements | 7,818,702 | 4,646,990 | 59.4 |
| 268 | Interest during construction | 920,730 | .... | .... |
| 274 | Miscellaneous construction expense[b] | .... | .... | .... |
| | Grand total | 192,934,549 | 86,572,304 | 44.87 |

[a] Negative net salvage; that is, cost of removing exceeds gross salvage. [b] In view of special character of this small class of property, and of meager amount of available actual experience data relative thereto, quantities for this item have been omitted.

by the advocates of the straight-line method when the accounting deals with property of hundreds of millions of dollars.

Table 3 brings out clearly the high percentages of salvage of telephone properties under the accounting plan of the telephone companies, higher probably than percentages to be found in any other utility industry due to the use, after restoration of many items of plant. This exhibit clearly shows the difficulty of the checking of accounts by people not familiar with these great properties.

Interest on the Credit Balance.—The credit balance in the reserve for retirement or so-called "depreciation" reflects money which has been collected from the ratepayers. Any excess over the total amount of retirements is usually invested in property additions. Mr. Vail stated in the 1913 American Telephone and Telegraph Company report that "all the present surplus and reserves is invested in * * * property * * * without paying capital charges on this

amount." At that time the credit balance was well in excess of 107 million dollars and saving of interest was in terms of millions of dollars per year whatever rate be used for the computation. If simple interest be computed on all the credit balances over the 30-yr period shown in Table 2, Col. 1, it will be found to be the equivalent of one year's interest on more than twenty-one billion dollars. No matter what rate of interest be used the resultant saving to the corporation of interest on money used for legitimate corporate uses, such as construction of new plant, is found to amount to hundreds of millions of dollars.

Apparently this question of interest saved to the owning corporation by the use of money reflected by the credit balance has not been made an issue in any of the telephone cases involving rates and valuation in the past, although it was suggested by the company in the Lindheimer Case "that an adjustment might be made by giving credit in favor of the telephone users in an amount equal to 3½% upon the difference between the depreciation reserve and the amount deducted from the valuation for existing depreciation." No matter what method is adopted in accounting the fact is inescapable that the credit balance in the reserve reflects money that has been advanced by ratepayers through charges to operating expenses in excess of needs; that its use in construction of plant obviates the securing of capital on which interest or dividends would be paid; and that excessive reserves not only constitute what Prof. H. W. Hatfield in his books on accounting terms "hidden surplus" in themselves, but secure to the corporation a substantial return in the form of saving of interest. This subject is again referred to later in this report. No emphasis is placed on this important matter in the NARUC report.

Early in this discussion of the straight-line reserves of the telephone company attention was directed to the increase of the credit balance in terms of percentage of fixed capital. From December 31, 1930, to December 31, 1940, during which time the credit balance increased from 19.68% of depreciable plant to 31.04%, the increase of the credit balance amounted to $636,131,000. In that same period the total investment in plant showed an increase of $706,437,000. Thus it appears that during these years of depression when all American business was seriously affected, the equivalent of 90% of all the money for extension of plant was derived from operating expenses through the depreciation reserve.

The Bell telephone industry developed the straight-line method in its accounting before railroad accounting came under regulation and before any regulation of other utilities. It fully and frankly discussed its system of accounting in its annual reports, pointing out matters that were open to criticism. The method has done all that Mr. Vail predicted for it and the results probably exceed any early expectations of the officers.

The company has defended its position in many lawsuits. It has been sustained in the perfectly sound position that money collected through rates belongs to the company and that property which reflects reserves created by money paid for rates, under commission approval, also belongs to the company. The Smith and Lindheimer cases, however, have held that excessive contributions to reserves should not be permitted to continue and the Lindheimer Case places the burden of proof on the company to show that they are not

excessive. As the law has been established by the opinions of the Court, the reserves are higher than needed for retirement and a reasonable balance to care for coming retirements. In the past they have served to build up an industry that has completely revolutionized the business and social communication system of the world, and through thorough research in this and other fields has perfected instrumentalities of service worth far more to the nation than any excess that has been paid by the patrons of the service.

The Bell Telephone Utility initiated and perfected its system of accounting just as the Congress approved public regulation of Railroad Accounting. As case after case involving utility rates and accounting practice has been decided by the Supreme Court a body of law has been built up which definitely limits the objectives of reserve accounting, and makes it very clear that the method used by the telephone and railroad utilities results in charges to operating expenses and the accumulation of reserve balances that are substantially in excess of permissible amounts.

## IX.—The Experience of the Railroads with Accounting by the Straight-Line Method

The railroads of the United States have been under the regulation of the Interstate Commerce Commission since 1887, but the authority of the Commission was not extended to the regulation of Accounting until 1907. At that time the late Dr. Henry C. Adams was "In charge of statistics and accounts" for the Commission. Professor Adams had been statistician for the Commission from its organization and was familiar with conditions on the carriers. He was associated as a consultant on the Michigan Valuations of Railroads in 1900 and 1903. He was much concerned at the very large deductions for depreciation which were made by this and other later State Valuations on a number of decadent little railroads and on the then dying lumber roads and branch lines of the large systems into the timber areas of the states of Michigan, Wisconsin, and Minnesota. The introduction of depreciation of equipment as an operating expense account in the first classification was largely due to Dr. Adams' concern regarding the failure of many companies to make any provision in accounting for recouping capital losses due to retirement of road and equipment. Many of the larger roads made a practice of charging replacements, including many large additions or betterments, to operating expenses. The regulation of accounting resulted in the abolition of many very serious evils. The weaker roads, however, were unable to set aside reserves to maintain value even though the state valuations disclosed serious amounts of actual depreciation or loss of service value.

Depreciation of equipment was provided for in the first classification of accounts. The second classification in 1914 provided for depreciation of way and structures, but owing to the strenuous opposition of the carriers the creation of reserves was made mandatory for equipment and optional for way and structures. In the third classification of 1936 depreciation for way and structures was again left optional with the carrier.

The straight-line method is specified in railroad accounting. On pages 88 and 89 of the 1936 classification the following excerpts are to be found:

"These charges for currently accruing depreciation shall be computed at such percentage rate of the ledger value of the equipment that the service value may be distributed under the straight-line method in equal annual charges to operating expenses and other accounts during the estimated service life of the equipment.

* * *

"(e) The carrier shall keep such records of equipment retirements as will reflect the service lives and value of salvage of each class of equipment; shall maintain in convenient and accessible form engineering and other data bearing on prospective service lives; and shall be prepared at any time upon direction of this commission to compute and submit for its approval, new percentage rates to take the place of those based on service lives or value of salvage found to be inaccurate."

The annual statistical reports of the Interstate Commerce Commission are among the most complete of all government statistics and furnish a wonderful record of railroad statistics and accounts for more than half a century but unfortunately do not permit a complete analysis of the accounting for depreciation until very recent years. Prior to 1934 the summaries for all Class 1 carriers do not separate, on the balance sheet, either investment or depreciation, as between road and structures and equipment. As many carriers do not charge depreciation to operating expenses for road and structures at all, and those that do make only very limited charges, the resulting percentage that the credit balance is of the investment furnishes no clue to the effect of the required straight-line accounting for equipment.

Table 4, derived from the Commission's Table 155, "Development of Steam Railways—1890–1942," shows the total investment in road and equipment for all United States railroads and total reserves for depreciation. This shows the fairly rapid growth of investment in railroads from 1911 to 1930 and the complete lack of growth from 1930 to 1942, the latter year showing less total investment than 1930. During these years the growth of the total credit balance from 1.35% in 1911 to 13.78% in 1942 gives no indication of the effect of the straight-line method adopted in accounting for equipment and the resulting percentages of credit balance to investment place the total reserves in the class of those of the electric companies, but do not represent the effect of any "method."

An analysis of the 1942 statistics shows that of a total of $3,232,242,347 in the credit balance of reserves of class I carriers only $171,249,504, or 5.3% of this total, was accrued for way and structures, and $3,060,992,843, or 94.7%, was accrued for depreciation of equipment.

Not until 1934 did the statistics of the Interstate Commerce Commission show in the summarized balance sheets for all class I carriers the separation of the investment on the asset side and of the reserve for depreciation on the liability side as between way and structures, and equipment. To make this separation and prepare a table showing the growth of the reserve for equipment would be a difficult task which the Society's committee would not be justified in undertaking, if it were possible to do so.

In 1926, in the consideration of Docket 15100, the Interstate Commerce

Commission had a study made of the depreciation of equipment from the reports of carriers which made the separation between road and equipment in their returns. This was a smaller group than that analyzed in Table 4, having 81.3% of the investment and 81.1% of the amount of money in the credit balance. This study furnishes a basis for showing the rate of growth of the

TABLE 4.—GROWTH OF DEPRECIATION RESERVES OF RAILROADS
(From Table 155, Interstate Commerce Commission Reports on Depreciation of Road and Equipment)

| Year ended | Investment in road and equipment book value (thousands of dollars) (1) | Depreciation Reserve; Road and Equipment | |
|---|---|---|---|
| | | Thousands of dollars (2) | Percentage of investment (3) |
| June 30: | | | |
| 1911 | 15,612,379 | 210,466 | 1.35 |
| 1912 | 16,004,745 | 259,661 | 1.62 |
| 1913 | 16,588,603 | 327,847 | 1.98 |
| 1914 | 17,153,786 | 435,835 | 2.54 |
| 1915 | 17,441,420 | 511,452 | 2.93 |
| 1916 | 17,689,425 | 571,360 | 3.23 |
| Dec. 31: | | | |
| 1916 | 17,842,777 | 628,934 | 3.52 |
| 1917 | 18,574,298 | 796,395 | 4.29 |
| 1918 | 18,984,756 | 936,979 | 4.93 |
| 1919 | 19,300,121 | 1,009,322 | 5.23 |
| 1920 | 19,849,320 | 1,081,342 | 5.44 |
| 1921 | 20,329,224 | 1,237,030 | 6.08 |
| 1922 | 20,580,001 | 1,335,453 | 6.50 |
| 1923 | 21,372,858 | 1,408,461 | 6.59 |
| 1924 | 22,182,267 | 1,547,969 | 6.98 |
| 1925 | 23,217,209 | 1,681,291 | 7.24 |
| 1926 | 23,880,740 | 1,811,002 | 7.58 |
| 1927 | 24,453,871 | 1,946,798 | 7.96 |
| 1928 | 24,875,984 | 2,043,976 | 8.22 |
| 1929 | 25,465,036 | 2,169,736 | 8.52 |
| 1930 | 26,051,000 | 2,360,767 | 9.06 |
| 1931 | 26,094,899 | 2,520,738 | 9.66 |
| 1932 | 26,086,991 | 2,632,922 | 10.09 |
| 1933 | 25,901,962 | 2,707,942 | 10.45 |
| 1934 | 25,681,608 | 2,764,726 | 10.75 |
| 1935 | 25,500,465 | 2,771,404 | 10.86 |
| 1936 | 25,432,388 | 2,809,063 | 11.45 |
| 1937 | 25,636,082 | 2,950,848 | 11.53 |
| 1938 | 25,595,739 | 3,044,972 | 11.89 |
| 1939 | 25,538,157 | 3,102,779 | 12.18 |
| 1940 | 25,646,014 | 3,095,237 | 12.10 |
| 1941 | 25,668,984 | 3,240,145 | 12.62 |
| 1942 | 25,838,351 | 3,561,570 | 13.78 |

credit balance from 1914 to 1923 (118 I.C.C. p. 335). The Interstate Commerce Commission statistics give the data for all class I carriers from 1934 to 1942.

From these sources Tables 5 and 6 have been prepared. The missing years 1924 to 1933, inclusive, in Table 5, would undoubtedly show a much more rapid increase in the credit balance after 1929 as from that date to 1936 the investment in equipment was practically stationary.

Analysis of Table 5 shows that investment in railway equipment has increased more than one billion dollars from 1936 to 1942 while during the same

period the credit balance increased $686,197,000, or enough to pay for two thirds of the increase.

Some advocates of the straight-line method contend that as a result of its use the reserve will ultimately reach 50% of the investment and remain substantially stationary. Table 5 shows that this condition has nearly been

TABLE 5.—GROWTH OF DEPRECIATION RESERVES OF RAILROADS[a]; DEPRECIATION OF EQUIPMENT

| Date[b] | Plant cost (thousands of dollars) | CREDIT BALANCE | |
|---|---|---|---|
| | | Thousands of dollars | Percentage of plant cost |
| June 30: | | | |
| 1914 | 1,475,589 | 243,589 | 16.48 |
| 1915 | 1,750,092 | 306,917 | 17.54 |
| Dec. 31: | | | |
| 1916 | 2,188,356 | 406,076 | 18.56 |
| 1917 | 2,462,635 | 539,074 | 21.89 |
| 1918 | 2,644,334 | 568,071 | 21.48 |
| 1919 | 2,749,519 | 631,647 | 22.97 |
| 1920 | 3,000,245 | 760,086 | 25.33 |
| 1921 | 3,076,335 | 874,659 | 28.43 |
| 1922 | 3,266,814 | 934,886 | 28.62 |
| 1923 | 3,967,271 | 1,074,892 | 27.09 |
| .... | .... | .... | .... |
| .... | .... | .... | .... |
| .... | .... | .... | .... |
| 1934 | 5,383,002 | 2,362,003 | 43.70 |
| 1935 | 5,269,885 | 2,369,273 | 44.96 |
| 1936 | 5,228,117 | 2,374,796 | 45.42 |
| 1937 | 5,483,350 | 2,504,131 | 45.67 |
| 1938 | 5,498,124 | 2,605,773 | 47.40 |
| 1939 | 5,475,784 | 2,661,811 | 48.61 |
| 1940 | 5,608,202 | 2,737,689 | 48.80 |
| 1941 | 5,869,545 | 2,866,570 | 48.84 |
| 1942 | 6,243,271 | 3,060,993 | 49.03 |

[a] Roads that report depreciation of equipment. [b] Years 1914 to 1923, inclusive, from 118 I.C.C. pp. 334 and 335. Years 1934 to 1942, inclusive, from Annual Statistics of Railroads, Interstate Commerce Commission.

TABLE 6.—CREDITS AND DEBITS; EQUIPMENT RESERVES FOR DEPRECIATION
(From I.C.C. Annual Statistical Reports)

| Year | Total credits to reserve | Percentage of investment | Total debits to reserve | Percentage of investment |
|---|---|---|---|---|
| 1938 | $199,893,533 | 3.63 | $ 98,250,949 | 1.80 |
| 1939 | 183,451,275 | 3.35 | 126,829,566 | 2.31 |
| 1940 | 187,680,918 | 3.34 | 111,802,662 | 1.99 |
| 1941 | 197,276,722 | 3.36 | 81,658,354 | 1.39 |
| 1942 | 262,501,070 | 4.20 | 60,022,716 | 0.96 |

reached with railway equipment. The statistics of the Interstate Commerce Commission, if thorough analysis were to be made, would hardly sustain the contention that the 49.03% credit balance in 1942 represents the physical condition of the equipment at that time. It is noted from Table 155 in the Interstate Commerce Commission statistics that in every year since 1911 there has been an increase in tractive power of locomotives and in capacity of freight cars, the present-day locomotive having 86% more power and the freight car

37% greater capacity. The old all-wooden cars have largely disappeared and steel or steel under frame cars have taken their place. It is further noted that the highest average traffic densities on all class I carriers between 1910 and 1940 were in the years 1926 to 1929, the highest being 1,732,295 ton miles per mile of road in 1926. Densities of 1,950,166 in 1941 and 2,638,067 in 1942 do not indicate a physical condition of equipment corresponding to the percentage shown for the reserve balance.

Examination of individual reports of carriers shows great differences in the percentages of the reserve for "depreciation" in the road accounts, ranging from nothing at all to 4% or 5%, but in the required accounting for equipment all of them show high percentages ranging from 45% to 49% and in a few cases of small roads with a limited amount of equipment the reserve exceeds 50%.

The most noticeable result of an analysis of these "straight-line" reserves is that, in periods of depression or other economic disturbance, the charges to operating expenses greatly exceed the retirements and actual accrued depreciation which would be deducted in valuation, and result in credit balances which reflect the collection of money from this source sufficient to defray a very large part of the cost of new construction. That is not the purpose for which depreciation reserves are set up.

To illustrate: (1) The reports of the Federal Communications Commission show that, from 1930 to 1938, the credit balance in the "depreciation" reserves of class A telephone companies increased $554,022,000, or 72.6%, while in the same period the total investment in plant of these companies increased $565,372,000 or 13.4%. In other words, the increase in the reserves provided more than 97% of the money needed for plant extension during this period of economic depression. (2) The reports of the Interstate Commerce Commission show that from 1930 to 1942, a period four years longer, the railroad plant account for all class I carriers decreased $212,650,000, or 0.81%, while the credit balance increased $1,200,803,000 or 50.8%. During the period there were many changes and great improvements in the equipment of the railroads of the nation.

The Society's committee points out these facts as they are reported by the commissions for the purpose of showing the results of straight-line accounting. Undoubtedly the money reflected in the reserve balances of the railroads went far to put them in condition to handle the greatest traffic in their history in the years immediately following this 11-yr period. However, financing plant additions is not the purpose for which depreciation charges are made.

The statistics that have been cited as to the two great industries, one of which may be said to have originated the straight-line method as a means of creating reserves for retirement, the other of which was compelled by the Interstate Commerce Commission to use it, are presented to substantiate this committee's belief that the straight-line method accrues substantially more than is a proper reserve for retirement of plant, and that it is not a simple but a very involved method which has been, and is, difficult and costly for commissions to regulate.

## X.—The Experience of Industries Which do not Use Straight-Line Method

*Water Works.—*

The Committee on Depreciation of the American Water Works Association (A.W.W.A.), in a study of 181 privately owned water works properties, reports for the year 1943 the following figures:

| | |
|---|---|
| Total depreciable property | $282,082,402 |
| Retirement reserves | 38,092,121 (13.4%) |
| Annual depreciation allowance | 2,820,540 (1.0%) |

Similar figures for 107 municipally owned water works are reported in the August 1944 Journal of the A.W.W.A. as follows: Total depreciable property $1,047,000,000, depreciation reserves equal to 27% of depreciable property and annual depreciation allowances of 1.77%. Publicly owned utilities commonly accrue excessive reserves as a deterrent to the diversion of profits to other channels and to provide funds required for new construction without the necessity of bond issues.

*Gas Companies.—*

The Committee on Depreciation of the American Gas Association has recently made a study of 72 privately owned properties.

The committee's study reveals the following facts:

| | |
|---|---|
| Total plant accounts | $6,650,836,000 |
| Retirement reserves exclusive of depletion | 1,018,701,000 (15.3% of plant cost) |

The Committee of the American Gas Association states that "The essential purpose of a retirement reserve is to provide a reserve adequate for the retirement of major units of property, not taken care of by maintenance, and to equalize large retirement losses over periods of years." This is an excellent statement of what is actually being done by most of the properties which do not use the straight-line method.

*Natural Gas, Pipe Lines, Etc.—*

The Society's committee has made no study of the practice of these utilities in the matter of reserves for depreciation or amortization. Gas reserves and in some cases the pipe lines from them are wasting assets. The problem with them is amortization rather than depreciation so far as the wasting assets are concerned.

*The Electric Power Utility.—*

The Electric Power Utility has used for many years the classification of the NARUC and, since 1937, the almost exactly similar classification of the Federal Power Commission. The NARUC Classification reads as follows:

> "Each utility shall record as at the end of each month the estimated amount of depreciation or amortization and depletion accrued during that month on utility plant subject to charges for depreciation, amortization and depletion."

The Federal Power Commission classification differs only by one word; the word "utility" plant is changed to read "electric" plant; and, on page 89, the classification provides for six subdivisions: Steam, Hydro and Internal Combustion Plants, and Transmission, Distribution, and General Plant.

This leaves the estimating of a proper and reasonable charge to the management of each company, without any restrictions as to the method of making

TABLE 7.—FEDERAL POWER COMMISSION DATA COVERING THE ENTIRE INDUSTRY

| Year | Total utility; book cost (thousands of dollars) | Reserve for Depreciation and Amortization of Utility | | Charge to Expense for Depreciation and Amortization | |
|---|---|---|---|---|---|
| | | Thousands of dollars | Percentage of total plant cost | Thousands of dollars | Percentage of total plant cost |
| 1937 | 13,851,583 | 1,495,250 | 10.80 | 212,865 | 1.53 |
| 1938 | 14,048,019 | 1,629,182 | 11.59 | 223,540 | 1.58 |
| 1939 | 14,114,695 | 1,762,386 | 12.48 | 243,178 | 1.70 |
| 1940 | 14,406,987 | 1,912,974 | 13.28 | 256,396 | 1.80 |
| 1941 | 14,736,793 | 2,096,412 | 14.22 | 274,913 | 1.87 |
| 1942 | 14,848,281 | 2,306,145 | 15.53 | 285,809 | 1.94 |
| Increase, 1942 over 1937: | | | | | |
| Dollars | 996,698 | 810,895 | .... | .... | .... |
| Percentage | 7.2 | 54.2 | .... | .... | .... |

TABLE 8.—RETIREMENT RESERVES AS REPORTED BY SEVENTEEN COMPANIES

| Year | Plant total (thousands of dollars) | Credit to Reserve | | Charges to Reserve | | Reserve Balance | |
|---|---|---|---|---|---|---|---|
| | | Total (thousands of dollars) | Percentage of plant | Total (thousands of dollars) | Percentage of plant | Total (thousands of dollars) | Percentage of plant |
| 1913 | 220,447 | 4,456 | 2.02 | 2,418 | 1.10 | 9,364 | 4.25 |
| 1914 | 245,547 | 4,595 | 1.87 | 1,369 | 0.56 | 12,696 | 5.17 |
| 1915 | 269,924 | 6,212 | 2.30 | 1,889 | 0.70 | 17,016 | 6.30 |
| 1916 | 293,649 | 6,636 | 2.27 | 3,196 | 1.09 | 20,903 | 7.12 |
| 1917 | 317,696 | 8,376 | 2.64 | 2,627 | 0.83 | 28,091 | 8.84 |
| 1918 | 449,952 | 7,961 | 1.77 | 4,216 | 0.94 | 34,395 | 7.64 |
| 1919 | 478,697 | 8,732 | 1.82 | 4,029 | 0.84 | 40,532 | 8.47 |
| 1920 | 522,686 | 11,257 | 2.15 | 5,374 | 1.03 | 47,327 | 9.05 |
| 1921 | 630,842 | 13,528 | 2.14 | 8,271 | 1.31 | 59,120 | 9.37 |
| 1922 | 713,638 | 16,084 | 2.25 | 8,972 | 1.26 | 63,752 | 8.93 |
| 1923 | 793,324 | 18,883 | 2.38 | 9,928 | 1.25 | 73,246 | 9.23 |
| 1924 | 917,109 | 20,846 | 2.27 | 11,603 | 1.27 | 85,285 | 9.30 |
| 1925 | 1,087,043 | 25,278 | 2.33 | 14,872 | 1.37 | 98,599 | 9.07 |
| 1926 | 1,232,047 | 29,825 | 2.42 | 17,444 | 1.42 | 112,804 | 9.16 |
| 1927 | 1,364,055 | 33,966 | 2.49 | 20,651 | 1.51 | 126,963 | 9.31 |
| 1928 | 1,534,366 | 38,403 | 2.50 | 22,861 | 1.49 | 143,393 | 9.35 |
| 1929 | 1,672,044 | 38,992 | 2.33 | 24,559 | 1.47 | 159,328 | 9.53 |
| 1930 | 1,855,641 | 37,802 | 2.04 | 28,697 | 1.55 | 170,587 | 9.19 |
| 1931 | 1,955,849 | 35,064 | 1.79 | 28,460 | 1.46 | 179,231 | 9.16 |
| 1932 | 2,017,912 | 35,962 | 1.78 | 18,391 | 0.91 | 200,460 | 9.93 |
| 1933 | 2,073,801 | 37,511 | 1.81 | 26,575 | 1.28 | 212,180 | 10.23 |
| 1934 | 2,084,583 | 41,029 | 1.97 | 28,859 | 1.38 | 226,176 | 10.85 |
| 1935 | 2,082,882 | 38,683 | 1.86 | 22,711 | 1.09 | 245,676 | 11.80 |
| 1936 | 2,103,320 | 49,682 | 2.36 | 22,491 | 1.05 | 275,456 | 13.10 |
| 1937 | 2,143,567 | 46,789 | 2.18 | 21,946 | 1.02 | 303,379 | 14.15 |
| 1938 | 2,231,943 | 53,007 | 2.37 | 23,485 | 1.05 | 341,220 | 15.29 |
| 1939 | 2,324,684 | 55,319 | 2.38 | 30,926 | 1.33 | 374,969 | 16.13 |
| 1940 | 2,358,328 | 57,745 | 2.45 | 33,805 | 1.43 | 400,555 | 16.98 |
| 1941 | 2,430,894 | 62,059 | 2.55 | 35,023 | 1.44 | 430,500 | 17.70 |
| 1942 | 2,493,766 | 66,052 | 2.65 | 25,253 | 1.01 | 472,953 | 18.97 |
| 1943 | 2,493,233 | 66,584 | 2.67 | 20,837 | 0.84 | 499,732 | 20.04 |

the estimate, but subject to the approval of the commission in control. This is a wise provision as there are great differences in the requirements of different companies in any industry, and also between industries. This provision is primarily a matter for determination by the management subject to the orders of the regulatory body. There now exist records under regulated accounting of from 25 to 30 years of actual retirements, salvage and other matters upon which most companies can base their estimates of their annual requirements

TABLE 9.—A MIDWESTERN POWER COMPANY; STATE-WIDE SERVICE
(All Figures Stated in Terms of Percentage of Book Value of Plant as of December 31st)

| Year[a] | Charges to operating expense for depreciation amortization | Total credits to reserve | Total debits to reserve | Excess of credits over debits | Credit balance in reserve |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| 1918 | 1.31 | 1.44 | 0.24 | 1.20 | 2.10 |
| 19 | 1.28 | 1.35 | 0.17 | 1.18 | 3.23 |
| 1920 | 1.15 | 1.23 | 0.57 | 0.66 | 3.57 |
| 21 | 1.65 | 1.75 | 1.25 | 0.51 | 3.92 |
| 22 | 1.43 | 2.12 | 0.60 | 1.51 | 4.56 |
| 23 | 1.43 | 1.55 | 1.06 | 0.49 | 4.60 |
| 24 | 1.48 | 1.64 | 1.43 | 0.22 | 4.24 |
| 1925 | 1.35 | 1.63 | 0.95 | 0.69 | 4.34 |
| 26 | 1.37 | 1.42 | 0.63 | 0.80 | 4.81 |
| 27 | 1.15 | 1.54 | 0.91 | 0.62 | 4.64 |
| 28 | 1.37 | 1.44 | 0.70 | 0.74 | 5.00 |
| 29 | 1.42 | 1.58 | 1.13 | 0.45 | 4.95 |
| 1930 | 1.48 | 1.95 | 1.17 | 0.78 | 5.06 |
| 31 | 1.41 | 1.51 | 1.11 | 0.40 | 5.19 |
| 32 | 1.38 | 1.44 | 0.70 | 0.73 | 5.84 |
| 33 | 1.37 | 1.60 | 1.13 | 0.47 | 6.26 |
| 34 | 1.36 | 2.62 | 1.70 | 0.92 | 6.98 |
| 1935 | 1.33 | 1.39 | 0.98 | 0.40 | 7.23 |
| 36 | 1.43 | 2.14 | 1.14 | 1.00 | 8.04 |
| 37 | 1.70 | 1.90 | 0.91 | 0.99 | 8.77 |
| 38 | 1.68 | 1.80 | 0.89 | 0.91 | 9.22 |
| 39 | 1.86 | 2.12 | 1.57 | 0.55 | 9.37 |
| 1940 | 1.99 | 2.14 | 1.60 | 0.54 | 9.61 |
| 41 | 2.44 | 2.60 | 2.45 | 0.16 | 9.63 |
| 42 | 2.82 | 3.06 | 1.70 | 1.36 | 10.90 |

[a] During this period, book value increased from approximately $50,000,000 to approximately $289,000,000, or 477%. The major part of this growth, 330%, to $215,000,000, was from 1918 to 1931; hence the more rapid growth of the reserve from 1931 to 1942. The 1937 cost was $247,200,000. Credit balances were as follows:

| Year | Balance |
|---|---|
| 1918 = | $ 1,050,000 |
| 1931 = | 11,158,500 |
| 1937 = | 21,679,000 |
| 1942 = | 31,501,000 |

which will maintain an adequate reserve. Since the year 1937 estimates have been subdivided as prescribed by the Federal Power Commission.

The Financial Record of the Electric Utility Industry 1937–1942, issued in December, 1943, by the Federal Power Commission, to the industry as a whole, gives the data presented in Table 7.

It will be noted from Table 7 that while the book cost was increasing 7.2% the credit balance increased 54.2%, again reflecting the rapid increase in the reserve balance in periods of slow growth of plant.

In order to get comparable figures with those of the straight-line method as used by the telephone industry for the whole period 1918 to 1942, the Society's committee has used the percentage figures developed in a study by the Edison Electric Institute of seventeen companies with a plant total of $2,493,233,000,

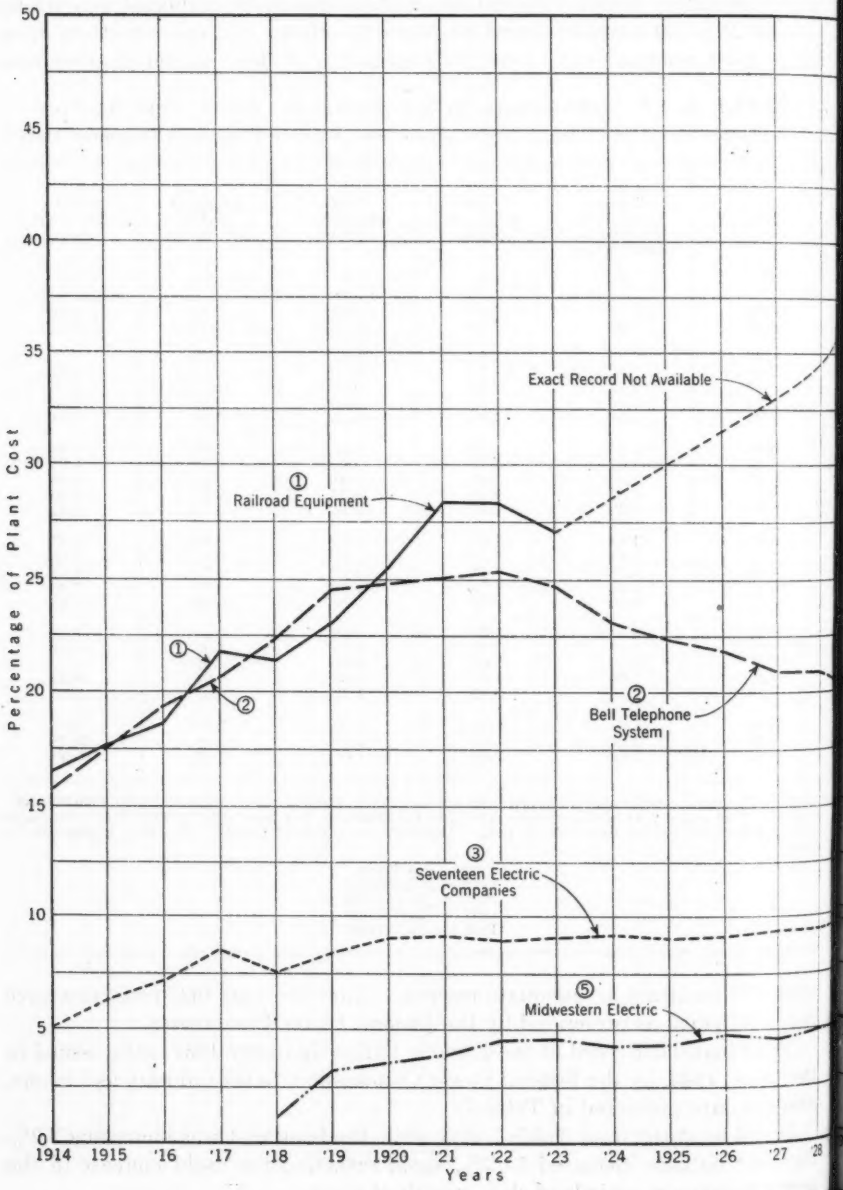
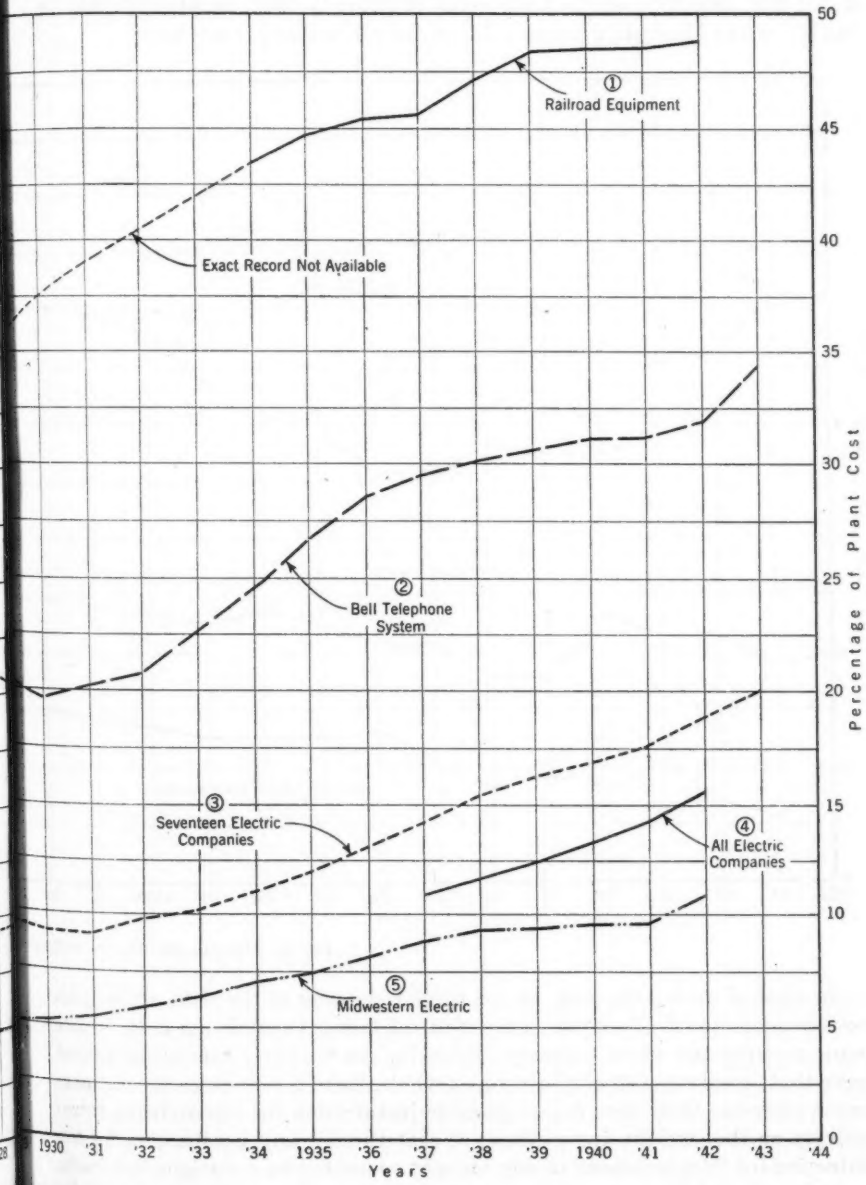



FIG. 1.—GROWTH OF CREDIT BAL

and also a study of a state-wide midwestern electric utility in the $300,000,000 class. These figures appear in Tables 8 and 9.

Table 8 shows the consolidated figures, in dollars and percentages of plant cost, the credits to the reserve (which in all cases are principally made up of




RESERVES FOR RETIREMENT

charges to operating expense for depreciation); debits to the reserve (net retirements); and credit balance in the reserve. It will be noted that in every year of the period credits exceeded debits in both dollars and percentage of total value of plant; consequently the credit balance in the reserve steadily increased from $9,364,000, or 4.25% of the book value of plant in 1913, to $499,732,000, or 20.04% of the immensely larger value of the plant thirty years later.

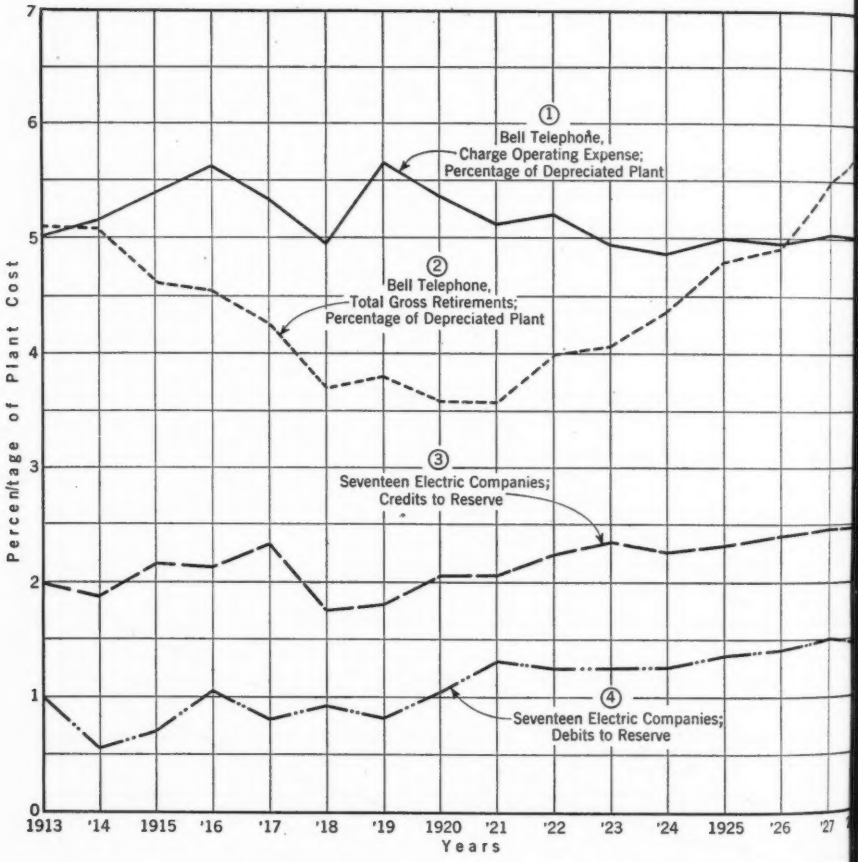
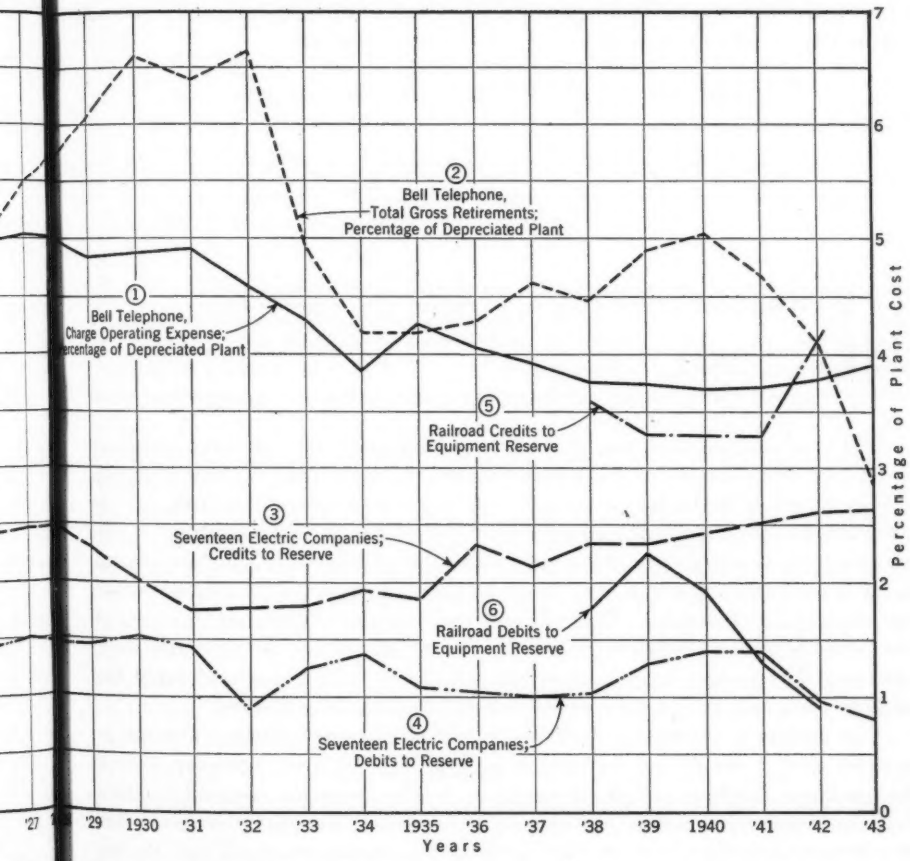



FIG. 2.—CREDITS AND DEBITS TO RESE

In view of such a showing, or the similar showing of the data of the mid-western single plant (Table 9), or the Federal Power Commission data for five years covering the whole industry (Table 7), the Society's committee cannot agree that, speaking of the industry as a whole, there is any necessity for more ample reserves; that there is any possible justification for recomputing retroactively on the straight-line method; or that there is any justification for the deduction for "depreciation" of any amount computed on a straight-line basis.

It is believed that these records may be accepted as fairly typical of the

results in the power industry of the method provided for in the present accounting classification. When the results from this far from complete study of existing records of the telephone and railroad equipment reserves treated by the use of the straight-line method and those of the Electric Corporation reserves, built up under the present NARUC accounting requirements, are compared in Figs. 1 and 2, it is perfectly evident that the straight-line method as used by


② Bell Telephone, Total Gross Retirements; Percentage of Depreciated Plant
① Bell Telephone, Charge Operating Expense; Percentage of Depreciated Plant
⑤ Railroad Credits to Equipment Reserve
③ Seventeen Electric Companies; Credits to Reserve
⑥ Railroad Debits to Equipment Reserve
④ Seventeen Electric Companies; Debits to Reserve
Percentage of Plant Cost
0
1
2
3
4
5
6
7
'29 1930 '31 '32 '33 '34 1935 '36 '37 '38 '39 1940 '41 '42 '43
Years

TERMS OF PERCENTAGE OF PLANT COST

the Telephone Utility and for railroad equipment results in credit balances very greatly in excess of those where judgment based on experience is the guide.

These studies indicate that, as an average, the utilities covered maintain adequate balances in the reserve, although some individual companies may have balances which are either too large or too small. When a method has been in use for 25 or 30 years and in not one year of the period have retirements and other debits to the reserve been sufficient to reduce the dollars in the credit balance—but on the other hand the credit balance has steadily grown year after

year, and unless it can be shown that the reserve is less than the amount of actual accrued depreciation—it cannot be said that the company is not making adequate provision "to keep the investment intact."

*Careful Analysis of Statistics Necessary.—*

The committee is fully convinced that Retirement Accounting, as now used by the electric, gas and water works industries, is in general providing credit balances in the reserves that are ample. Table 10 has been prepared to summarize and compare some of the figures already given.

TABLE 10.—SUMMARY AND COMPARISON OF DATA

| Year | Utility | Investment in plant | Credit Balance in Reserve | | Method of computing depreciation |
|---|---|---|---|---|---|
| | | | Dollars | Percentage of investment | |
| 1943 | Telephone | 4,990,000,000 | 1,713,175,000 | 34.34 | Straight line |
| 1941 | Railroad (equipment) | 5,752,904,000 | 2,467,735,000 | 42.9 | Straight line |
| 1943 | Gas | 6,650,836,000 | 1,018,701,000 | 15.3 | Other than straight line |
| 1942 | Electric | 14,848,000,000 | 2,306,000,000 | 15.5 | Other than straight line |
| 1943 | Water works (private) | .... | .... | 13.4 | Other than straight line |
| 1943 | Water works (public) | .... | .... | 27.0 | Predominantly straight line |

The total amount of money that has been collected from ratepayers over and above past retirements that have been charged to "depreciation," now probably well in excess of eight billion dollars, for all these groups of utilities, directs attention to the magnitude and importance of this form of accounting. Only the railroads, the largest in investment of all the utilities (and which in general apply it only on equipment), and many of the publicly owned water works use the straight-line formula. These figures are doubtless somewhat in excess of the total amount needed for retirement reserves. It must be clearly understood, however, that a very large part of these balances are proper and fulfill the purpose for which they were created and approved by the Courts.

The Society's committee does not condemn the straight-line formula or contend that it should not be used in accounting. It does, however, believe that on large, far-flung utilities it results in the accumulation of credit balances that are substantially above any needed requirement, and that, due to the fact that its accumulations have no relation to actual depreciation, and that the use of it tends to cause managements to consider the reserve a mere matter of formula and not to give this important subject sufficient attention, its use should not be required.

Those who advocate the use of the straight-line method usually adopt average lives in its application rather than those derived from a special study of the mortality of units on the particular property, and without any check of the results of its application in the relationship between the size of the reserve and the actual depreciation in the property. This may and often does result in wide variation from the truth, and in gross inequity.

## XI.—Reconsideration of "Depreciation" is Essential Now

There is unquestionably a present need for a full and complete discussion of the subject of accounting methods for creating reserves for plant retirement which is the maximum permissible size of those reserves, and for the final determination of the relationship of such reserves, actual depreciation, and the amount to be deducted in the determination of a rate base.

In connection with his discussion of Dockets 14700 and 15100 which dealt with proposed changes in telephone and railroad accounting, Mr. Eastman said, in November, 1926 (118 I.C.C. on p. 356):

> "It is essential to bear in mind, however, that there is an inseparable connection between the straight-line method and the principle that accrued depreciation represented by the depreciation reserve must be deducted in ascertaining the rate base value. If any other principle should hereafter be adopted by the courts a reconsideration of the entire question of depreciation would at once become necessary."

It is quite probable that this statement of eighteen years ago was prompted by the opinion of the Supreme Court in the New Jersey Telephone Case (271 U.S. 23) decided earlier in that year, which held squarely that the property which was represented by the reserve belonged to the company, just as much so as if it had been acquired through the sale of stocks and bonds. Obviously the doctrine stated by the Court in that case and later reaffirmed by Chief Justice Hughes in Smith vs. Illinois Bell Telephone Company (282 U.S. 133) is in direct contradiction of the theory stated by Commissioner Eastman in the statement just quoted. These Supreme Court opinions clearly prohibit the deduction of reserves which have been built up under regulated accounting. The subsequent opinion in the Lindheimer Case (292 U.S. 151), handed down in 1934 by Chief Justice Hughes, discussed the straight-line method at length. A study of this long opinion, which was a unanimous opinion of the Court, expanded somewhat in a concurring opinion of Mr. Justice Butler, makes it very clear that it is in no sense an approval of the straight-line method.

The Society's committee believes that the evident meaning of the Court is not to be found in any single paragraph and therefore includes a rather extended quotation from this most important opinion:

> "Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence. Annual depreciation is the loss that takes place in a year. In determining reasonable rates for supplying public service, it is proper to include in the operating expenses, that is in the cost of producing the service, an allowance for the consumption of capital in order to maintain the integrity of the investment in the service rendered. The amount necessary to be provided annually for this purpose is the subject of estimate and computation. In this instance, the Company has used the 'straight-line' method of computation, a method approved by the Interstate Commerce Commission. 177 I.C.C. pp. 408, 413. By this method the annual depreciation charge is obtained by dividing the estimated service value by the number of years of estimated service life. The method is designed to spread evenly over the service life of the property the loss which is realized when the property is ultimately retired from service.

"As the allowances for depreciation, credited to the depreciation reserve account, are charged to operating expenses, the depreciation reserve invested in the property thus represents, at a given time, the amount of the investment which has been made out of the proceeds of telephone rates for the ostensible purpose of replacing capital consumed. If the predictions of service life were entirely accurate and retirements were made when and as these predictions were precisely fulfilled, the depreciation reserve would represent the consumption of capital, on a cost basis, according to the method which spreads that loss over the respective service periods. But if the amounts charged to operating expenses and credited to the account for depreciation reserve are excessive, to that extent subscribers for the telephone service are required to provide, in effect, capital contributions, not to make good losses incurred by the utility in the service rendered and thus to keep its investment unimpaired, but to secure additional plant and equipment upon which the utility expects a return.

* * *

"The calculations are mathematical but the predictions underlying them are essentially matters of opinion. They proceed from studies of the 'behavior of large groups' of items. These studies are beset with a host of perplexing problems. Their determination involves the examination of many variable elements and opportunities for excessive allowances, even under a correct system of accounting, are always present. The necessity of checking the results is not questioned. The predictions must meet the controlling test of experience.

* * *

"In this instance, the evidence of expert computations of the amounts required for annual allowances does not stand alone. In striking contrast is the proof of the actual condition of the plant as maintained—proof which the Company strongly emphasizes as complete and indisputable in its sharp criticism of the amount of accrued depreciation found by the District Court in valuing the property. The Company insists that 'the existing depreciation in the property physical and functional, does not exceed 9 per cent in the years 1923 to 1928 and 8 per cent thereafter.' The existing depreciation as thus asserted by the Company, and the amounts it shows as the depreciation reserve allocated to the intrastate business in Chicago (taking in each case the average amounts per year) are as follows:

| Years | Existing depreciation | Depreciation reserve |
|---|---|---|
| 1923 | $11,992,000 | $26,797,000 |
| 1924 | 12,865,000 | 29,316,000 |
| 1925 | 13,775,000 | 32,155,000 |
| 1926 | 14,621,000 | 35,572,000 |
| 1927 | 15,360,000 | 39,352,000 |
| 1928 | 16,241,000 | 42,769,000 |
| 1929 | 15,300,000 | 44,515,000 |
| 1930 | 15,863,000 | 45,829,000 |
| 1931 | 15,828,000 | 48,362,000 |

* * *

"Giving full weight to these considerations, we are not persuaded that they are adequate to explain the great disparity which the evidence reveals. As the Company's Counsel says: 'The reserve balance and the actual depreciation at any time can be compared only after examining the property to ascertain its condition; the depreciation, physical and functional, thus found can be measured in dollars and the amount compared with the reserve.' Here we are dealing not simply with a particular year but with a period of many years—a fairly long range of experience—and with careful

and detailed examinations made both at the beginning and near the end of that period. The showing of the condition of the property, and of the way in which it has been maintained, put the matter in a strong light."

* * *

"This condition, kept at a nearly constant level, directs attention to the amounts expended for current maintenance. In the process of current maintenance, 'new parts' are 'installed to replace old parts' in units of property not retired. Such 'substitutions or 'repair'' are separate from the amounts which figure in the depreciation reserve. The distinction between expenses for current maintenance and depreciation is theoretically clear. Depreciation is defined as the expense occasioned by the using up of physical property employed as fixed capita; current maintenance, as the expense occasioned in keeping the physical property in the condition required for continued use during its service life. But it is evident that the distinction is a difficult one to observe in practice with scientific precision, and that outlays for maintenance charged to current expenses may involve many substitutions of new for old parts which tend to keep down the accrued depreciation."

* * *

"In the light of the evidence as to the expenditures for current maintenance and the proved condition of the property—in the face of the disparity between the actual extent of depreciation, as ascertained according to the comprehensive standards used by the Company's witnesses, and the amount of the depreciation reserve—it cannot be said that the Company has established that the reserve merely represents the consumption of capital in the service rendered. Rather it appears that the depreciation reserve to a large extent represents provision for capital additions, over and above the amount required to cover capital consumption. This excess in the balance of the reserve account has been built up by excessive annual allowances for depreciation charged to operating expenses.

"* * * The point is as to the necessity for the annual charges for depreciation, as made or claimed by the Company, in order to avoid confiscation through the rates in suit. On that point the Company has the burden of proof. We find that this burden has not been sustained. Nor is the result changed by figuring the allowances at the somewhat reduced amounts fixed by the court below.

"We find this point to be a critical one. The questionable amounts annually charged to operating expenses for depreciation are large enough to destroy any basis for holding that it has been convincingly shown that the reduction in income through the rates in suit would produce confiscation."

The Supreme Court reversed the Court below, dissolved the injunction, ordered a refunding of the amounts charged by the Company in excess of the rates in suit, and dismissed the bill of complaint.

As the Society's committee interprets this opinion it squarely holds that the straight-line reserves were excessive; that the straight-line method is "beset with a host of perplexing problems * * * and opportunities for excessive allowances * * * are always present"; that "the predictions must meet the controlling test of experience"; that the engineering determination of actual depreciation in the case was most convincing.

In a concurring opinion Mr. Justice Butler said:

"The Company's properties constitute a complex and highly developed instrumentality containing many classes of items that require renewal from time to time. But, taken as a whole, the plant must be deemed to be permanent. It never was intended to be new in all its parts. It would be impossible to make it so. Expenditures in an attempt to accomplish that

would be wasteful. Amounts sufficient to create a reserve balance that is the same percentage of total cost of depreciable items as their age is of their total service life cannot be accepted as legitimate additions to operating expenses. In the absence of proof definitely establishing what annual deductions from revenues were necessary for adequate maintenance of the property, the company is not entitled to have the rate order set aside as confiscatory."

The Society's committee is unable to read into the opinion in the Lindheimer Case, or any of the preceding cases back to 1909, any suggestion or inference that a reserve designed to return to the company, through charges to operating expenses, the money to make good losses due to retirement, or even to cover the theoretical consumption of capital, is to be deducted from value. The Lindheimer opinion of Chief Justice Hughes, and the concurring opinion of Mr. Justice Butler, emphasize the fact that the straight-line method results in reserves grossly exceeding actual depreciation.

To what extent the opinion of the Hope Natural Gas Case, decided January 4, 1944, will affect utility regulation or prescribed accounting remains to be seen. That case dealt primarily with the extent of authority of the Commission under the Natural Gas Act, where a 40-yr old company was being brought under regulation for the first time. The facts of the case as stated in Mr. Justice Douglas' opinion clearly appear to justify the conclusions of the Commission, which found the "actual legitimate cost" of the property to be approximately $52,000,000 and deducted as depreciation $22,328,000, or 43%, in the process of determining a rate base. The company had accumulated a credit balance in the reserve of about $46,000,000, and in the presentation of its own case deducted about $30,000,000 as depreciation, apparently based on a present value computation. It should be noted that these figures involve both depletion of Natural Gas facilities a wasting resource (an amortization problem) as well as depreciation of physical property, two distinctly separate things.

Mr. Justice Douglas said:

"In determining the amount of accrued depletion and depreciation the Commission, following Lindheimer v. Illinois Bell Tel. Co. 292 U. S. 151, * * * based its computation on 'actual legitimate cost.' It found that Hope during the years when its business was not under regulation did not observe 'sound depreciation and depletion practices' but 'actually accumulated an excessive reserve of about $46,000,000.' * * * One member of the Commission thought that the entire amount of the reserve should be deducted from 'actual legitimate cost' in determining the rate base. The majority of the Commission concluded, however, that where, as here, a business is brought under regulation for the first time and where incorrect depreciation and depletion practices have prevailed, the deduction of the reserve requirement (actual existing depreciation and depletion) rather than the excessive reserve should be made so as to lay a sound basis for future regulation and control of rates."

It is very clear on a study of this case that it is wholly unlike any that would be likely to occur where a property had been under regulation for a quarter of a century.

The deduction was an amount less than half of the credit balance in the reserve, less in dollars than the depreciation estimated by the Company, and certainly affords no precedent for the deduction of a credit balance established

under regulated accounting. It does however emphasize in a most positive manner the warning of the Lindheimer and other cases against excessive reserves.

"To keep the investment unimpaired" is not to provide a reserve to be deducted from value in a rate base. That would be just plain confiscation. But any reserve that accumulates more than enough to provide for return to the company cost of plant that has been and is to be retired is taking from the consumers money to be invested in capital which should be furnished by stockholders or through the issue of bonds. It is very difficult to understand why the NARUC committee in its report recommends the straight-line method which results in the accumulation of vast and obviously excessive reserves at the expense of the consumers.

Referring once more to Commissioner Eastman's statement at the beginning of this section of the report, it seems to be evident that the adoption of the straight-line method was, in his mind, inseparably connected with the deduction of the accumulated credit balances in the "Depreciation" reserve as accrued depreciation in ascertaining a rate base value. He called for a reconsideration of the "entire question of depreciation" if the Court decided otherwise. In the same month in which Mr. Eastman wrote the statement quoted, which was probably prompted by the opinion in the New York Telephone Case (271 U.S. 23), John W. McCardle et al. vs. Indianapolis Water Company (272 U.S. 400) was decided, the opinion being handed down by Mr. Justice Butler on November 23, 1926. In this opinion are the following statements:

> "There is deducted approximately 25% of estimated cost new to cover accrued depreciation. The deduction was not based on an inspection of the property. It was the result of a straight-line calculation based on age and the estimated or assumed useful life of the perishable elements. The Commission's report indicates that property is well planned, well maintained and efficient."
>
> * * *
>
> "The testimony of competent valuation engineers who examined the property and made estimates in respect of its condition is to be preferred to mere calculations based on averages and assumed probabilities. The deduction made in the City's estimate cannot be approved. Pacific Gas Co. v. San Francisco, 265 U.S. 403–406; Standard Oil Co. v. Southern Pacific, Supra 159; Landon v. Court of Industrial Relations, 269 Fed. 433–445; City of Winona v. Wisconsin Minnesota Light and Power Co. 276 Fed., 996–1004; New York Telephone Co. v. Prendergast, 300 Fed., 822, 826; Southern Bell Tel. & Tel. Co. vs. Railroad Commission, 5 Fed. 2nd 77–95."

This opinion makes it perfectly evident that there was a long line of precedent approving the deduction of observed depreciation as found on an engineering study of property. That case apparently should have caused abandonment of the belief that the straight-line computation of accounting reserves for retirement was a proper figure for accrued depreciation to be deducted in a rate procedure.

In the eighteen years that have elapsed since then the concept has been clung to by some of the men in regulatory bodies. Ten years ago Mr. Chief Justice Hughes made clear the fact that the purpose of reserve accounting is to maintain the integrity of the investment, not to destroy it, and discussed the straight-

line method at length, pointing out its weaknesses, comparing the credit balance in the reserve with the accrued depreciation found by engineering inspection and definitely approving the inspection method.

In spite of his apparently definite opinion in the Lindheimer Case, which was approvingly cited in the Hope Natural Gas Case in 1944 (subsequent to the NARUC committee's report), we find, on page xix of the NARUC committee's report, in No. 40, the following statement: "In fixing public utility rates adequate depreciation expense should be allowed accordingly to the service life basis and, in principle, the reserve requirement determined on the same basis should be deducted in determining the rate base."

With this principle the Society's committee is in complete disagreement for the reasons stated at length in this section of this report.

## XII.—The Engineering Determination of Accrued Depreciation

The engineering determination of accrued depreciation referred to and recommended in the Progress Report of the Society's committee[1] is a determination made by competent and reliable individuals, who, after a study of the property involved, determine what the actual effect of depreciation has been up to some specific date. Such a determination includes the consideration of many factual data, upon which an opinion as to the amount of accrued depreciation is finally based. As stated in the progress report, this method has in the past been referred to as the "Observation" method, and the results styled as "Observed Depreciation." Opponents of this method have capitalized on the word observed, and have tried to belittle it by inferring that the determination was made by simply observing or looking over the property. Such may have been and probably was the case in some instances. However, there have been undoubtedly just as many instances where the straight-line method has been used, where the only basis was a life table in some textbook applied wholly without consideration of the property whose depreciation was being determined.

A proper engineering determination of accrued depreciation includes the consideration of all data available in regard to the unit of property, and is the sort of determination made by most responsible and experienced men engaged in valuation work.

The process of making an engineering determination of accrued depreciation varies somewhat from property to property and among engineers who are experienced in appraisal work. Essentially, it includes the following steps, not necessarily in this chronological order.

The books and records of the property are examined with the assistance of employees of the company charged with the responsibility of keeping records, and who are familiar with them. From these records there is prepared a list of property units, or groups of like units, and, if available, the dates when they were installed, their cost new, installed, ready for service, including appropriate overhead costs (engineering, general, and interest lost during construction) and the expenses for maintenance and repairs during the period from installation to date.

The engineer then makes an inspection of the units, accompanied by such assistants as may be required, examining each unit of property, observing its

conditions during operation, determining its operation behavior from inquiry from operators, and ascertaining what repairs may be imminent, and any other factors bearing upon the character and extent of deferred maintenance. In many cases machines are partially dismantled to permit examination of working parts. In the case of static property, such as pipe, wire, poles, transformers and the like, the examination includes a notation of corrosion, decay, abnormal characteristics developed during service, etc.

Included in the study is an analysis of past and probable future growth of the system and the city as affecting the relationship of the size of unit and adaptability for continued future service. The character of the community is studied with respect to its type; that is, whether residential or industrial, etc., and any indication of impending changes in the character of the load. Load curves of the system are also helpful in the consideration of the adaptability of the unit in question, to serve with other units, the size of which also may have a bearing upon the probable place of the particular unit in the future operating schedule.

It will also be of assistance if mortality statistics for the property are assembled from the records of retirements whenever records permit. These data may be arranged in the form of curves or diagrams. Other pertinent mortality data are examined and where applicable to the local conditions are assembled in the form of mortality curves if sufficient applicable data are available to permit doing so. A study of type mortality curves and the selection of those adapted to the local conditions and the units of property will be helpful in interpreting such data as may be otherwise available and will assist in arriving at a final judgment as to future service by the unit.

It will frequently, perhaps usually, be found that no pertinent mortality data are available on the particular property or type of unit being studied and the experienced engineer should be very careful to use only such types of mortality curves as he feels certain are applicable.

From such a study it is practicable for the engineer to reach a conclusion as to the probable future usefulness of the unit. In the light of all the data relative to the unit, its operations, capacity for future service, probable future usefulness, the consideration of whatever mortality statistics may be applicable, and the use of any mortality curves applicable to the particular unit, the engineer can then arrive at the percentage condition of the property and the total accrued depreciation which is the ultimate end desired.

The Society's committee believes that an engineering determination of accrued depreciation, made as outlined above, is the only way the actual effect of depreciation can be obtained, and that accounting practice should be so planned that a reserve for depreciation sufficient to meet the requirements indicated by such a determination will result. This belief is in marked contrast to that expressed in the recommendations of the NARUC committee, that annual allowances for depreciation should be calculated by the straight-line method and that the reserve created by such allowances becomes the measure of actual depreciation.

It is believed that if those connected with public utility regulation would accept the fact that the effect of depreciation on utility property has been, is now, and probably always will be subject to many variables, differing in each

property, and that it cannot be determined by any theoretical uniform system of accounting a lot of the fog that has been thrown around the subject would burn off. It is proper to use theoretical methods in the accounting for annual allowances, and to create a reserve on that basis, but such a reserve would seldom, if ever, constitute a fair measure of accrued depreciation. The annual allowances should be adjusted to meet the requirements by periodic determinations of what the accrued depreciation actually is. The statement of Mr. Justice Jackson in the Hope Natural Gas Case is well worth repeating—it expresses the danger in strict adherence to fixed accounting conventions as follows:

> "To make a fetish of mere accounting is to shield from examination the deeper causes, forces, movements, and conditions which should govern rates. Even as a recording of current transactions, bookkeeping is hardly an exact science. As a representation of the conditions and trend of a business, it uses symbols of certainty to express values that actually are in constant flux. It may be said that in commercial or investment banking or any business extending credit success depends on knowing what not to believe in accounting. Few concerns go into bankruptcy or reorganization whose books do not show them solvent and often even profitable. If one cannot rely on accountancy accurately to disclose past or current conditions of a business, the fallacy of using it as a sole guide to future price policy ought to be apparent. However, our quest for certitude is so ardent that we pay an irrational reverence to a technique which uses symbols of certainty, even though experience again and again warns that they are delusive."

## XIII.—Retroactive Adjustment of the Depreciation Reserve

The NARUC Committee on Depreciation in its report recommends that "The reserve should be deducted * * * in arriving at the rate base for the reserve alone measures actual depreciation." The NARUC Depreciation Committee also recommends that, where the depreciation reserve as computed by the straight-line method materially exceeds the reserve currently in the property, an adjustment shall be made in the amount necessary to bring the current reserve up to that computed on the straight-line basis; this adjustment to be made through charge to surplus or special section of the income account.

Before proceeding to discuss these recommendations it is well to review the fundamental concept of depreciation. Depreciation is an operating cost. Even the NARUC committee recognizes this in their statement: "Depreciation results in a cost of service." Mr. Justice Moody, in the Knoxville water case stated it admirably as follows:

> "Before coming to the question of profit at all, the company is entitled to earn a sufficient sum annually to provide, not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. * * * The Company is entitled to see that from earnings the value of the property invested is kept unimpaired, so that at the end of any given term of years the original investment remains as it was in the beginning."

It is apparent from this excerpt from the Knoxville decision which has been the primary judicial basis for depreciation allowances for nearly forty years, that the investor is entitled to see that the value of the property is kept unimpaired. How can the value be kept unimpaired if, as the NARUC committee

now recommends where the credit balance in the depreciation reserve is less than the amount computed on a straight-line, age-life basis, the difference be withdrawn from surplus or capital? If that subtraction is made, as recommended by the NARUC committee, the capital is directly impaired. By that method an unearned operating expense becomes a deduction from capital.

The unfairness of this recommendation is further shown when it is recalled that practically all utilities have now been under supervision of public regulatory bodies for the past twenty to forty years. During that period their depreciation allowances have either been determined or approved by these regulatory bodies. To now arbitrarily require capital adjustments to bring the results of public regulatory policies up to the measurements of straight-line age-life depreciation, and to reflect this adjustment in the valuation or rate base cannot, in the opinion of the Society's committee, be viewed in any other way than the taking of property without the process of law.

In the Columbus Gas and Fuel Company vs. Public Utilities Commission of Ohio case decided May 21, 1934, in an opinion delivered by Mr. Justice Cardozo, the following statement appears:

> "To withhold from a public utility the privilege of including a depletion allowance among its operating expenses, while confining it to a return of 6½% upon the value of its wasting assets, is to take its property away from it without due process of law, at least where the waste is inevitable and rapid * * *. Plainly the state must either surrender the power to limit the return or else concede to the business a compensating privilege to preserve its capital intact."

Justice Cardozo then quotes from the Knoxville decision:

> "Before coming to the question of profit at all the company is entitled to earn a sufficient sum annually to provide not only for current repairs, but for making good the depreciation and replacing parts of the property when they come to the end of their life."

He then continued:

> "It is idle to argue that a company using up its capital in the operations of the year will have received the same return as one that at the end of the year has its capital intact and interest besides."

This quotation is identical with the statement appearing in the Dayton Power and Light case decided April 30, 1934.

The telephone utilities and railroads stand alone in their exclusive use of the straight-line method of figuring annual depreciation. As a result they have accumulated in many instances reserves materially larger than the actual depreciation in the property.

Practically all other utilities have depreciation reserves which currently amount to approximately one half to one third of the reserves that would be computed on a straight-line basis. It is with this great number of properties that the retroactive adjustment of the depreciation reserve is so serious. It is, however, most significant that in these telephone cases of excessive reserves the Court reduced the annual allowance rather than declaring the excessive reserve to be "actual" depreciation.

The NARUC committee report realizes that the retroactive adjustment is a serious problem but it nevertheless makes it even more serious when it states:

> "The depreciation reserve should not be readjusted gradually through modification of the annual depreciation rates when the difference between the book reserves and proper reserve (straight-line) is substantial."

With most utilities this difference is substantial, probably averaging 100% or more of book reserves. The stipulation above quoted closes the door to the only means of collecting depreciation reserves from the consumers whose service used up the property. Since the utility is precluded from a gradual readjustment through collections from the consumers, the only alternative necessarily results in impairment of its property. To require the utilities to deduct from capital amounts which they have allowed to collect through rates under regulation is contrary to all precedent and ethical procedure.

It is of interest to note that the recommendation of the NARUC committee on retroactive accounting is not new. In 1926, in the original order in Docket No. 15100, the Interstate Commerce Commission, Commissioner Eastman handing down the opinion, said (118 I.C.C. on p. 384):

> "Past depreciation unprovided for. In the case of steam railroads, and perhaps to some extent in the case of telephone companies, a difficulty which will be encountered is that as a rule depreciation accounting has not in the past been observed with respect to fixed property. In such cases there is no depreciation reserve which now covers that part of the service value of existing units which has already been consumed. The theoretically correct way of meeting this situation would be to establish the amount of past accrued depreciation which has not been provided for, and concurrently credit this amount to the depreciation reserve and charge it to profit and loss. It is the latter account which has profited in the past from the failure or partial failure to accrue depreciation charges. In view of the fact that such procedure might, in some and perhaps in many instances, result in converting a profit and loss surplus into a deficit with a consequent impairment of credit, it seems desirable that for the present the amount so credited to the depreciation reserve should be concurrently charged to a suspense account on the assets side of the balance sheet, instead of to profit and loss. This suspense account could then be gradually extinguished by charges to profit and loss over a maximum period of years or in such shorter time as the carrier might elect."

This ruling, which deducted approximately 20% of the total cost of reproduction found by the Commission from the basic valuation which was to be used in establishing the new accounting, was criticized by the railroads at the subsequent hearings. There were many objections but the principal one was that, in many instances, the effect of disregarding former accounting and setting up the "past accrued depreciation," as developed by the formula of the Commission, would be to seriously decrease or wipe out the profit and loss surplus, and to seriously impair the credit of all railroads. The device of a suspense account did not in any way relieve the railroad from the burden of liquidating these enormous sums through profit and loss and was merely a makeshift.

In the Proposed Report, dated August 15, 1929, the Commission substantially changed its position and in all the succeeding years has never placed retroactive depreciation accounting into effect.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# FAILURE OF THE TACOMA NARROWS BRIDGE

## REPORT OF THE SPECIAL COMMITTEE OF THE BOARD OF DIRECTION

### Discussion

BY LOUIS BALOG

LOUIS BALOG,[8] ESQ.[8a]—Because this report is the only record that is easily available to any one interested in the official investigations made and opinions expressed concerning the causes of this instructive failure, some critical comment on the views it contains may well be made. Besides the material presented in this report, many words were poured into the wake of the disaster in various periodicals partly listed in Appendix V. The characteristic feature of the report and of many of these articles is that they are critical without contributing to the solution of the problem in question; and they leave the structural eminence of the bridge unnoticed. The articles which suggest blind limitations for the dimensional ratios of suspended structures, or convey false information as to the effectiveness of the so-called "very economical corrective measures" do harm by minimizing the necessity of appropriations for experimental research with the aid of which the design of safe, economical, and well-appearing structures can be attained.

In abstracting the reports the committee concentrates on the statical, and entirely neglects to review the aerodynamic, investigations that these first researches into suspension bridge behavior would have well deserved. Instead of giving such a much-needed review to dispel the faulty conceptions created by the mass of misleading information written and spoken following the failure, the committee attributes statements of similar character to Report (*a*) which evidently are not contained therein (see Appendix I). Whereas aerodynamicists recognized the lack of knowledge concerning the intricate problem involved, civil engineers emphasize aerodynamical considerations which do not

NOTE.—This report was published in December, 1943, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: April, 1944, by Francis P. Witmer; and June, 1944, by Homer M. Hadley.

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[8a] Received by the Secretary August 29, 1944.

apply to the case. Disregarding the contents of Report (*a*), Professor Cissel (52)[8b] takes designers to task for the "lamentable lag * * * to apply sound aerodynamic principles"; but what those principles were at the time of the design he fails to indicate. (Rigid stiffening trusses do not follow from aerodynamic principles which modify shape only; trusses are a way out of, but not a solution to, the problem.) Study of Report (*a*) would have shown that from the point of view of practical design such sound principles did not exist (and have not since been established), and would have saved the committee's few and meager comments from being in contradiction to the little aerodynamical knowledge gained by the reported investigations.

Describing the structure, the committee states (see heading, "General Features of the Bridge Project: Dimensions and Composition") that the stiffening girders "were exceptionally shallow and consequently had little vertical rigidity." Shallow girders were aerodynamically more advantageous than deeper ones. The increase of rigidity that results from the use of deeper girders, within practical limits, cannot offset the great aerodynamic disadvantages of increased depth. Had the girders been still shallower by the amount of that part above the deck, the bridge might possibly be standing today stabilized by a modification of the suspended structure. This is evident from the investigations presented in Report (*a*), from the results of the experiments at the University of Washington, and from the behavior of actual structures.

Based on comparative investigations of girders 8 ft and 13 ft deep, Report (*a*) states (page 122):

> "The evidence for occurrence of negative damping in the case of vertical oscillations of the Tacoma Bridge is at least questionable. However, there is definite evidence that deep plate girders can cause negative damping in vertical oscillations * * *."

To this it must be added that girders 13 ft deep represent a 63% increase in static wind loads and require an increase in the girder, girder bracing, lateral bracing, tower and cable metal, and anchorage concrete, amounting in all to $500,000 additional cost. The moment of inertia of the girders would double. Report (*a*) clearly shows (Fig. 20, page 90) that the corresponding increase in stiffness is negligible.

The objection to the design because it did not include details with "sufficient capacity to dissipate energy" (Appendix II of the Society's report) is incapable of proof without (*a*) practical suggestions for energy-dissipating details of sufficient capacity; and (*b*) a measure for their efficiency. Energy-dissipating splices are impractical; floor details with a frictional by-product have proved themselves inefficient on the George Washington Bridge. This bridge has the greatest static stiffness vertically and torsionally according to the classification of structures in Report (*a*) and Report (*c*); and in addition its floor structure, nearly 120 ft wide, is the most articulated of any actual bridge. The stringers, fascia girders, four lines of substantial double-step type steel curbs and handrails, roadway slabs, and sidewalk slabs are provided with expansion connec-

[8b] Numerals in parentheses, thus: (52), refer to corresponding items in the Bibliography (see Appendix V) of the report, and at the end of discussion in this issue.

tions at every floor beam at distances of half the width of the floor. Aerodynamically, its suspended structure has a comparatively advantageous shape; yet at times the bridge has oscillated in wind sufficiently to cause discomfort to the users. The true magnitude and character of the movements of this bridge have not been made public.

Any other but a rigid connection of structural elements is either faulty or is a necessary evil from the structural point of view. That even the most carefully constructed models grossly overestimate the effect of small frictional resistances is indicated by the differences between the Bronx-Whitestone Bridge model and prototype behavior in this respect. Friction due to small displacements in the joints and floor structure may change considerably during the life of the bridge and stability should not depend on such friction even if its effect were otherwise not negligible. Fundamentally sound structural principles should not be abandoned for imaginary advantages before it is proved that other considerations outweigh their value.

The committee (see Appendix I) calls the comparison of deflections of various bridges due to 200 lb per ft of bridge given in Report (*a*) and subsequently in Report (*c*) of "little value * * * due to the various widths of the bridges" and states: "Had the comparison been for a uniform load per foot of cable, the results would have been quite different." This objection is not correct. The deflections given are for a uniform load of 100 lb per ft of cable and they are not affected by the width.

As to the value of these comparative deflections in the design as customarily calculated, static quantities do not give a scale for the relative rigidity of the structures. The data on five bridges, given in Table 2 (which for the Tacoma Narrows Bridge are based on 50% larger live load than was used in the design), clearly indicate this fact. The deflections of the quarter point due to the same uniform load from tower to mid-span, however, will furnish a scale of comparative rigidity if this load for all bridges results in a sufficiently small value of the $(p/w)$-parameter. Considering the prevailing dead loads, 100 lb per ft of cable was found to satisfy this requirement.

These comparative deflections resulting from 100 lb per ft of cable half-span load gave the first published study for a useful static classification of suspension bridges. (It should be remembered that the change in grade due to live load was the accepted measure of comparative rigidity of suspension bridges prior to the failure (70*a*)(70*b*)(70*c*)(70*d*)(70*e*)(70*f*).) This study began with the attempt to compare the vertical and torsional rigidities of the Bronx-Whitestone and Tacoma bridges long before the failure, and was extended by the writer for a large number of actual bridges and designs. This investigation included the comparative deflections of the unstiffened cables and the percentage contribution of the cables and stiffening system, side spans, and towers to the total rigidity, being the basis of correlation of static considerations with dynamic behavior as given in Report (*a*). Although the writer does not attach undue importance to these comparative data, the committee's "quite different" values cannot give a true picture. This is indicated by the fact that from the cable and girder stiffness parameters, $\frac{l^2}{H}$ and $\frac{l^4}{E\,I}$, comparative rigidities derived

by the writer result in the same quantitative sequence as those derived from the deflections. An objection similar to that raised by the committee concerning the consideration of the width was also raised by D. B. Steinman, M. Am. Soc. C. E. (67), in 1941. Mr. Steinman changed his mind two years later, however, and stated that "* * * the comparative rigidity of suspension bridges is gaged by such deflections" (71).

That the usefulness of these comparative deflections was underestimated by the committee is also indicated by the fact that Shortridge Hardesty, M. Am. Soc. C. E., in coupling such deflections with empirical considerations derived a dependable method for the determination of the required moment of inertia of stiffening trusses in 1941. By establishing an empirical connection between stability and the stiffness parameters referred to in the foregoing, the writer found that trusses having a total moment of inertia (inches$^2$-feet$^2$) of $0.005\,l^2$, or $0.0009\,H_w$ ($l$ in feet, $H$ in pounds), assure stability within the range of actual structures. Any attempt to derive the required rigidity from purely empirical considerations, however, should be regarded as a temporary expedient for estimating purposes and should not take the place of specific experimental research for each structure to be designed. Such research always pays for itself and advances general knowledge.

The committee's statement (see heading, "Committee Conclusions") that "* * * the methods of design computations * * * were not satisfactory for designing a structure of the proportions used in the Tacoma Narrows Bridge" is misleading. In fact, the methods of design computations used are, and will be, employed in the future for the design of similar structures with the irrelevant difference that methods applying the trigonometric series in calculating cable stresses, deflections, and moments may be substituted for the method operating with exponential functions used for such calculations in this case. The conclusion should be rather that the design computations were incomplete because of the omission of aerodynamic considerations. To this must be added, however, the statement that the behavior of existing structures, of even more slender proportions than this bridge, did not indicate this necessity; and the generally accepted beliefs and practice excluded the possibility of going beyond the customary consideration of the effects of wind loads. At the time of the design the only aid the designer could have was a practical theory which was nonexistent then (as it is still nonexistent), by which the magnitude and character of the aerodynamic loads that the structure was called upon to bear could be calculated and the shape of the suspended structure modified for their elimination.

In criticizing the design methods, therefore, the committee members should have examined the governing ideas of suspension bridge design at the time of the design and, acting as representatives of a professional Society, they should have given consideration to the conditions of professional practice that influence suspension bridge design. Such nontechnical factors were responsible for the strange fact that the facilities of modern research were not utilized, and are not being utilized, to broaden knowledge and improve design.

A review of the Society's publications concerning the design of suspension bridges since the construction of the George Washington Bridge reveals that the

governing ideas of design were regarded as having reached perfection. In this connection the following quotations are of interest:

(1) "Suspension structures * * * are always in stable equilibrium for any loading. * * * This anciently known fact early led to the construction of important suspension bridges (600-ft spans over the Menai Strait * * * in 1832, and over the Danube at Budapest, Hungary, in 1843) stiffened only nominally with hand-rails. Nothing serious can happen to a suspension bridge as long as the anchorage, towers, and cables are sound" (70*g*).

(2) "This important and governing fact [Item 1] should be kept in mind in discussing any further refinements in stiffening truss analysis" (70*h*).

(3) "Earlier concepts * * * placed too much emphasis on a mistaken interpretation of the function of the stiffening truss. * * * If the cable does not need help, the stiffening truss may well be omitted * * *" (70*a*).

(4) "* * * change in gradient is a * * * criterion of comparative rigidity [70*d*]. Criteria of comparing and specifying the rigidity of suspension structures have been introduced" (70*f*).

(5) "It is now well established that long-span suspension bridges for modern highway traffic may have a relatively flexible stiffening system, and that the degree of flexibility has a material effect upon the economy of design. * * * A depth of 11 ft for the stiffening girders [of the Bronx-Whitestone Bridge] was found to be sufficient * * *" (72).

(6) "* * * the cables relieve the stiffening trusses of wind load moments * * *. These increments of relief are so substantial that, in long-span work, they silence all arguments for unnecessary floor widths, heavy wind framing, * * * or other devices such as lateral stay cables * * * [39*a*]. This advantage becomes especially valuable * * * when the possibilities of wind and other vibrations are considered" (39*b*).

The foregoing quotations were based on considerations of structural elastic stiffness and resulted in the following generally accepted scales of rigidity:

(*a*) The measure of vertical stiffness is the maximum change in grade due to live load;

(*b*) The measure of lateral stiffness is the lateral angular change of roadway alinement at the towers due to a horizontal wind load; and

(*c*) The measure of torsional stiffness is the maximum transverse angular change of the roadway due to live load which is antisymmetrical with respect to both longitudinal and transverse center lines of the bridge.

These scales were the cooperative product of the engineers engaged in suspension bridge design and were regarded as representing the summation of all experience with suspension bridges. In a long-span suspension bridge that has been properly laid out, with either a steel or aluminum-alloy suspended structure, and any type of flooring, the generally accepted rigidity requirements, gaged by the foregoing scales, could be satisfied by the dead load alone. Following the layout of the suspended structure and the determination of proper span ratios, with due consideration to pier and anchorage foundations, compliance with the rigidity requirements could be achieved by manipulation of the sag in the cables. This design procedure results in structures of great economy and beauty. Nor can any fault be found with this procedure except for the importance ascribed to the calculated deflections and the misleading conclusions derived from them. Nothing did more harm to the proper de-

velopment of suspension bridge design than the exaggerated acclaim with which the paper "Suspension Bridges Under the Action of Lateral Forces" (39) was received and the claims resulting from it (39*a*)(39*b*)(39*c*)(39*d*)(40). Because a static test agreed with the calculated deflections, observation of the behavior of structures actually built and experimental research on the effect of wind loads were regarded as superfluous in the fully developed science of suspension bridge design.

Commenting on the methods of design the committee might have emphasized that it has been standard practice for a long time to carry out concurrently the design and laboratory model testing of hydraulic and aeronautical structures. These laboratories were recognized as self-evident necessities and have been directly connected with the design of less important structures. The behavior of the Thousand Islands Bridge, the first of the oscillating structures, should have initiated the proper wind-tunnel investigations, which neither the design nor even the oscillations of the subsequent large suspension bridges could inspire. Lesser events than the failure of the Tacoma Bridge could not spur the introduction of suitable experimental research such as has been simultaneous with design as an everyday practice in other branches of engineering.

The reason for this inertia lay in the conditions of the bridge designing profession where progress could be attained only if it did not entail increase in the cost of the design. Therefore, any advance depended entirely on the ability and personal inclination of the designer to use his own time for research and to evaluate and apply available research made for other purposes. The problem of aerodynamic stability, however, was far beyond the possibility of solution by such means. The meager knowledge resulting from theoretical study and from experimental research of aerodynamicists undertaken with extensive apparatus, prior to and after the failure, bears testimony to this, "* * * in spite of the fact that much attention has been given to the study of aeronautics in recent years."

Examination of Report (*a*) will show that it does not contain the statements the committee assigns to it in the text following conclusion 15, but quite the contrary. This fact discloses a deplorable attitude, the motives of which the writer fails to grasp. An extension of the investigations, necessary for independent judgment, could not reasonably be expected from the committee, but it could be expected that it will not present statements out of agreement with facts and make them appear as if they were to be found in the official report.

It should be clearly understood that thorough official investigation resulted in the conclusion that the bridge was well designed and in full conformity with accepted rigidity standards. Its flexibility was not apparent by these standards according to which it was one of the stiffest bridges built, its change of grade due to design live load (2.4%) was smaller than that of any of the bridges listed in Table 2, except for the George Washington Bridge. This measure of rigidity and the magnitude of design live loads were extensively discussed in the Society's publications and were adopted by the profession as the result of long years of experience in suspension bridge construction; and the behavior of the

oscillating bridges was not recognized as following from the deficiency of this rigidity scale prior to the investigations presented in Report (*a*). It was disclosed that structural elastic stiffness does not necessarily assure aerodynamic stability which follows from sufficient structural or aerodynamic damping; this, however, became known only from these investigations and should not be made to appear as if it was known at the time of the design.

Professor Farquharson had been observing the bridge and experimenting with its dynamic and wind-tunnel models for half a year, when he wrote by letter on October 29, 1940:

> "* * * it was observed that quite violent motions developing double amplitudes in the neighborhood of 4 ft could develop with winds less than 10 miles per hour. * * * To our very great surprise we found that in the ten miles an hour region the lift forces on the bridge probably never exceeded 2 lb per linear ft of the prototype."

If it had been possible to arrive at this result by calculations at the time of the design with the aid of material which, according to the committee (see Appendix I), "* * * is well known and is the subject of analytical treatment in textbooks dealing with mechanical vibrations," why did half a year's study with two models result only in a "very great surprise" two years later?

If aeronautical research and textbooks on vibrations, available previous to the design of this bridge, include information that could have guided the layout of its suspended structure, then why is it that experimental and theoretical research have failed to show any useful results up to now; and why did the committee approve the appointment of a national committee of twenty five to study this problem, instead of giving useful references in aeronautical and vibration textbooks?

The textbook the committee cites (62*a*) states: "* * * which cross sections are dynamically stable and which are unstable * * * brings us into the domain of aerodynamics, a science which is unfortunately still very little developed"; and (62*b*) "The values of lift and drag of an arbitrary cross section cannot be calculated from theory but can be found from wind-tunnel test." The suggestion of making wind-tunnel, or any other, tests was out of the question in 1938, and when made in 1940–1941 they prompted the statements that: "* * * for the specific cross section used in the Tacoma Bridge the evidence of instability is not conclusive" (Report (*a*), page 128); and further that "The aerodynamic investigations * * * give definite evidence of self-induced torsional oscillations for * * * practically all cross sections used or proposed for the design of suspension bridges * * *" (Report (*a*), page 123).

Even after the failure, available observations and results of experimental research were insufficient for a theoretical treatment of the problem. This fact is well demonstrated by a paper published in 1943 (73). The author, Hans Reissner, was the first to offer rigorous solutions for many difficult problems in all branches of mechanics. The results of the application of Professor Reissner's theory to a large number of actual bridges, however, are contrary to the facts of experience.

Perusal of Chapter I of Report (*a*) reveals five years of negotiations for a Tacoma bridge, in great contrast to the time spent with its design. Preliminary

studies—the conception of the structure and its design—were made by three engineers (Frederick Lienhard, M. Am. Soc. C. E., Ezra G. Odley, and the writer), in a five-week period during July and August, 1938. (The writer also made the investigations presented in Chapter II and Chapter III of Report (*a*).) This group possessed what constituted knowledge in suspension bridge design at that time, except for the information that the newly completed Thousand Islands Bridge oscillated in the wind. Although publications described this structure and also the oscillating bridges built subsequently, the fact that the bridges oscillated was not published anywhere at that time. The first publication disclosing the oscillations of the Thousand Islands, Deer Isle, and Bronx-Whitestone bridges (47) appeared a month after the Tacoma Bridge failed, stating that they were stabilized by the devices described. As a matter of fact, the bridges continued oscillating.

When the first oscillations of the Bronx-Whitestone Bridge occurred early in 1939, the character of the motion indicated that the long-known damping principles—of (1) tying the center of the cable to the towers, of (2) tying the center of the cable to the girder with rigid members or strands (74)(75), or of (3) a combination of the two (that is, connecting the cable to the girder at the center and the ends of the girder to the towers)—would be helpful. Consideration was given to connecting the cable to the towers by diagonal stays, as proposed in 1873 by Sir Thomas Bouch (76), and as done during the past twenty years during the construction of large suspension bridges. Also considered was the connection of the girder to the tower with diagonal stays. Although such installations have proved themselves inefficient in the past (77), their small expense justified a check on them by model experimentation. A carefully constructed model, however, overestimated the effectiveness of these devices (78). This fact and the statement that the engineers for the Thousand Islands and Deer Isle bridges had eliminated even the "tendency" for oscillation of these structures with such devices before they were completed, for "less than $5,000," diverted the experimental research begun at long last for the Tacoma Narrows Bridge into the wrong direction and caused a loss of valuable time for wind-tunnel investigations.

The wind-tunnel investigators were the first to advance knowledge beyond century-old misconceptions which, after two years of observation of the oscillating bridges, still prevailed. They looked for the cause of the oscillations in "gusty wind impulses," "cambered roadway," and "lateral flexibility," and tried to combat oscillations by static means. The wind tunnel showed that the oscillations were induced by the unsuspected aerodynamic characteristics of the suspended structures, indicating the proper way for their elimination and the necessity of combination research and design. In a few years this procedure resulted in the great achievements of aircraft designers; still it could not find its way into the old art of suspension bridge design.

To do justice to the Tacoma Bridge it should be noted that it was designed with the full utilization of the available results of theoretical and experimental research assembled to that date to improve upon the structural design of aircraft, bridges, and ships. It was the first example of the general application of the thin, stiffened plate as a basic structural element on a really large scale and

it set a new standard for the best practice in structural design. The economic progress resulting from this step deserves to be mentioned. For example: One tower of the Bronx-Whitestone Bridge, only one year older and supporting only a 40% larger floor area and a smaller static wind load, contains as much metal as the two towers of the Tacoma Bridge, which are 67 ft higher. Applying the same design principles to the towers of the Golden Gate Bridge, three years older, the savings would have been considerable, because, in further development of this design principle, the tower shaft sections required for the direct stresses could take care of the bending stresses without any additional metal.

The behavior of the structure under the terrific loads it received during the collapse has proved the soundness of the structural design methods. As a full-scale test, it created confidence in stiffened plate construction resulting in its general adoption in the design of large structures to great economical, erectional, and esthetic advantage. The monetary loss caused by the collapse will be regained many times by the savings made in other stiffened plate structures following this pioneering example.

It is sad indeed that a structure of the clean-cut slender elegance of the Tacoma Narrows Bridge had to perish. The loss is intensified by the fact that its failure, up to now, has resulted in the return to evolutionary solutions (deep stiffening trusses) in suspension bridge design. The writer shares the committee's hope that the research inspired by the collapse will be successful in solving the aerodynamic problem involved, thus making possible the safe use of "ribbon-like," well-appearing suspended structures fitting the character of this bridge system. In that case the Tacoma Narrows Bridge, in its passing, has done more for the progress in suspension bridge design, and in more than one way, than if it had carried traffic for a century.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# ORIGINAL AND READJUSTED REPAYMENT CONTRACTS, BOULDER CANYON PROJECT

## Discussion

BY CLAY C. ELDER

CLAY C. ELDER,[9] M. AM. SOC. C. E.[9a]—The discussions by Messrs. Chadwick and Arledge are helpful in presenting the viewpoints of the two generating agencies at Boulder Dam. Their clarifying statements are greatly appreciated. To one who recalls the controversies that raged around articles on Boulder Dam a decade or two ago, when the project was being planned, authorized, and constructed, the absence of similar present discussions is at first surprising. This lack of controversy indicates at least that the Adjustment Act of 1941 has been a practical success, in spite of its numerous compromises, and that the compromises have left no serious ill feeling among the seven states and numerous interests involved. The Adjustment Act relates chiefly to rate revisions and revenue allocations, however. On more fundamental questions, the lack of present criticism seems to indicate a successful accomplishment of the project's multiple purposes—flood control, water conservation, power development, silt control, water quality improvement, recreational uses, and even the improvement of navigation.

With the closing of the Boulder Dam magnesium plant, as recently announced, the project is leading the way into the period of reconversion, even as it was previously in the forefront of total war production. The Boulder Dam power output and demand are not decreased by the change but regionally an important saving of essential oil and gas fuel results from the decreased operation of steam standby plants.

The only remaining serious controversies involving the Boulder Canyon Project relate to the allocation of permanent water rights to its conserved flood waters. Recent ratification by Arizona of the Colorado River Compact

NOTE.—This paper by Clay C. Elder was published in December, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1944, by W. L. Chadwick; and February, 1944, by A. R. Arledge.

[9] Hydr. Engr., Met. Water Dist. of Southern California, Los Angeles, Calif.

[9a] Received by the Secretary October 9, 1944.

seemed a major step toward real peace, although current proposals for large irrigation diversions in Arizona above Boulder Dam may finally cancel many of the benefits of the Adjustment Act and may make the project insolvent as to cost repayment to the federal government. Of more immediate importance is the pending treaty with Mexico relating to the utilization of the waters of the Rio Grande and the Colorado River. Hearings on this proposed treaty are scheduled for early in 1945 before the Senate Foreign Relations Committee and, judging from the preliminary rounds of the battle, will be as hotly contested as any similar arguments of the "good old days." So at Boulder Dam, and along the Colorado River, as elsewhere in the world, it appears that "peace in our times" has yet to be achieved.

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Founded November 5, 1852

DISCUSSIONS

# ANALYSIS OF RIGID FRAMES BY SUPERPOSITION

## Discussion

BY DAVID B. HALL, A. A. EREMIN, AND VICTOR R. BERGMAN

DAVID B. HALL,[13] ASSOC. M. AM. SOC. C. E.[13a]—A simple orderly procedure for analyzing rigid frames with sidesway is presented in this paper. It is instructive to observe that relations exist between step 1, in which the geometrical properties of the cells are established, and step 4, in which conditions of equilibrium are established. It is the purpose of this discussion to point out these relations and to show that in the case of a single cell nonrectangular quadrangular frame they may be of considerable use.

In any structure the geometrical properties and static properties are closely interrelated through the law of conservation of energy. Thus, in most cases geometric methods and static methods are very similar, but because the relationship is an inverse one there will be occasions when the two are not equally simple. For example, the static analysis of a truss by vector diagram is simpler than the geometric analysis by Williot diagram because in the former all lines are parallel to members, whereas in the latter there are lines both parallel and perpendicular to members.

In the case of the quadrangular frame, an examination of the paper discloses that the statical analysis (Eqs. 22 to 25) requires a piecemeal method of approach, as contrasted with the simple geometric relation (Eq. 20). This difference appears to be characteristic of quadrangular cells. It seems logical, therefore, to apply the law of conservation of energy to geometric relations for the static analysis of the problem, just as it has been found advantageous to apply this law (as the method of virtual work) to find deflections of trusses from statical relations.

Before proceeding to do this, it will be advantageous to reestablish the geometrical relations in a manner slightly different from Eq. 20. When one side of a quadrangular cell remains fixed (AD, Fig. 29), the ends of the opposite member (BC) are constrained to move at right angles to their respective

NOTE.—This paper by David M. Wilson was published in February, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1944, by F. S. Merritt, Ralph W. Stewart, and John E. Goldberg; June, 1944, by A. W. Fischer, Leon Blog, Arthur B. McGee, and Jaroslav Polivka; and September, 1944, by Odd Albert, and W. C. Spiker.

[13] Stress Analyst, Otis Elevator Co., New York, N. Y.

[13a] Received by the Secretary September 1, 1944.

connecting members (AB and CD), and the member rotates about an instantaneous center located at the intersection of the connecting members (point O). In this connection it may be convenient to visualize triangle BCO

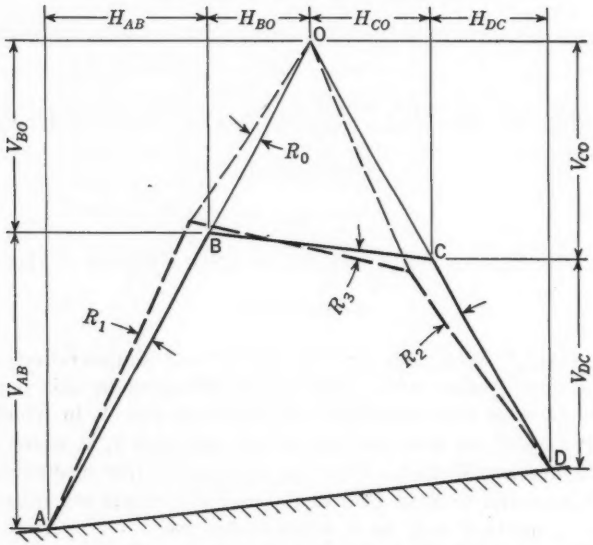


FIG. 29.—DEFORMATION OF A QUADRANGLE REFERRED TO INSTANTANEOUS CENTER

as an imaginary rigid body supported on the two legs. If this imaginary body is given a rotation $R_0$, then:

$$R_3 = R_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82a)$$

$$R_1 = -R_0 \frac{\overline{BO}}{\overline{AB}} = -R_0 \frac{V_{BO}}{V_{AB}} = -R_0 \frac{H_{BO}}{H_{AB}} \ldots\ldots\ldots\ldots (82b)$$

and

$$R_2 = -R_0 \frac{\overline{CO}}{\overline{DC}} = -R_0 \frac{V_{CO}}{V_{DC}}$$

$$= -R_0 \frac{H_{CO}}{H_{DC}} \ldots\ldots\ldots\ldots (82c)$$

Thus, the rotations of the various members are quite simply expressed in terms of a single parameter $R_0$.

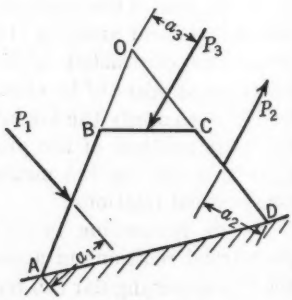


FIG. 30.—NOTATION FOR LOADS ON A QUADRANGULAR FRAME

Energy equations can now be written. For notation relating to loads refer to Fig. 30. Apply a rotation $R_0$ at the instantaneous center. The work done by loads on member BC equals $P_3 a_3 R_0$. The work done by loads on the legs equals $P_1 a_1 R_1$ and $P_2 a_2 R_2$. The work done by moments at A and D equals $M_{AB} R_1$ and $M_{DC} R_2$. The work done by moments at B and C equals

$M_{BA}\,(R_1 - R_3)$ and $M_{CD}\,(R_2 - R_3)$. Summing up the work done by all items and dividing by $R_0$,

$$P_3\,a_3 + P_1\,a_1\frac{R_1}{R_0} + P_2\,a_2\frac{R_2}{R_0} + M_{AB}\frac{R_1}{R_0} + M_{DC}\frac{R_2}{R_0} + M_{BA}\left(\frac{R_1}{R_0} - 1\right) + M_{CD}\left(\frac{R_2}{R_0} - 1\right) = 0 \quad \ldots\ldots\ldots\ldots (83)$$

As an example this method will be applied to the trapezoidal frame shown in Fig. 8(*a*). The legs intersect on the center line 15 ft above BC— $-R_1 = -R_2 = R_3 = R_0$. There are two loads of the type $P_3$ and no others; thus—

$$\begin{aligned} -200 \text{ lb} \times 15 &= -3{,}000 \\ 1{,}000 \text{ lb} \times 2.5 &= \phantom{-}2{,}500 \\ \hline \Sigma P_3\,a_3 &= -\phantom{3}500 \end{aligned}$$

and

$$-500 - M_{AB} - M_{DC} - 2\,M_{BA} - 2\,M_{CD} = 0 \quad \ldots\ldots\ldots\ldots (84)$$

Values of moments in terms of loads and unknown displacements are given in Fig. 8(*b*) and Fig. 8(*c*). In Fig. 8(*c*) it would be more orderly, when using the proposed method, to change all signs and refer the moments to $R_0$, but this is not necessary. Substituting the values of moments from Fig. 8(*b*) and Fig. 8(*c*) in Eq. 84, the following equation results:

$$\begin{aligned} &-500 - (194 - 6.46\,E\,R_1) - (-338 - 6.46\,E\,R_1) \\ &- 2\,(391 - 6.92\,E\,R_1) - 2\,(-677 - 6.92\,E\,R_1) = 0 \quad \ldots\ldots\ldots (85) \end{aligned}$$

Solving Eq. 85, $E\,R_1 = -5.32$, which agrees with the author's value.

After Eq. 83 has been derived by energy methods, it is not very difficult to retrace the solution and discover the significance of Eq. 83 as an equilibrium equation: Each term of the equation can be shown to represent a moment about point O, and the equation therefore is a statement that the net moment about point O is equal to zero. Thus, the equation could have been obtained without introducing energy, but its discovery would not have been so obvious and its derivation would not have been so simple.

A. A. EREMIN,[14] ASSOC. M. AM. SOC. C. E.[14a]—An interesting method of applying the principle of superposition to the algebraic distribution of moments in rigid frames is presented in this paper. The same results can be obtained, with less effort, by applying superposition principles to the graphical distribution of moments. The moment diagram derived by graphical distribution offers a means of comparing the directions of moments and forces visually, and obviates the need for approximate sketching, such as that suggested by the author. Furthermore, graphic methods can be checked readily, simply by inspection.

[14] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[14a] Received by the Secretary September 30, 1944.

For example, consider the frame in Fig. 31(*a*) in which the cross sections are constant. Basic stiffnesses, $\frac{I}{L}$, are shown in circles. The moments are determined graphically by the use of "three-line polygons" and "cross lines" (23)[14b] constructed at the "central points," points of inflection, and elastic points (see Fig. 31(*b*)) of each member.

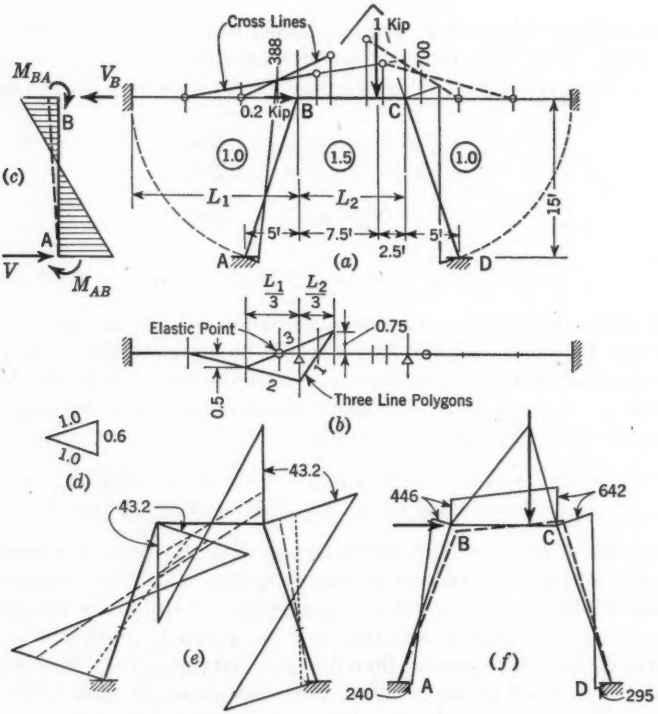


FIG. 31.—GRAPHICAL SOLUTION BY SUPERPOSITION

When all members of the frame have uniform cross sections the "central points" are at the third points of the span, and the point of inflection is located by drawing line 1, Fig. 31(*b*), through the stiffness ordinate at the support and through the ordinate at the third point that marks one half the basic stiffness. The stiffness ordinate for each member is determined by drawing line 2, Fig. 31(*b*), through the point of inflection at the far end and through the ordinate at the third point that marks one half the basic stiffness of the span being analyzed. The point of inflection in the member with a fixed end is at the third point of the span. The elastic point for the intermediate joint B, Fig. 31(*a*), is determined by constructing a three-line polygon. Line 3, Fig. 31(*b*), is drawn through the intersection of line 2 with the vertical at the third

[14b] Numerals in parentheses, thus: (23), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

point of the span and through the intersection of line 1 with the vertical at the third point in the adjacent span. The place where line 3, Fig. 31(*b*), intersects the axis of the member is the elastic point.

Moments due to the vertical force are determined graphically by "cross lines" (see Fig. 31(*a*)). When joint B is displaced a unit distance the resulting distortion diagram is as shown in Fig. 31(*d*); and the resulting moments in member AB, by the formula—

$$M_{AB} = \frac{600\, I\, a}{L^2_1\,(L_1 - a - b)} \ldots\ldots\ldots\ldots\ldots\ldots (86)$$

—are $M_{AB} = 29.6$ and $M_{BA} = 21.3$. In Eq. 86, $a$ and $b$ are the distances of the points of inflection from ends A and B, respectively. The numerical factor 600 is introduced for convenience of computation.

The manner of distributing the moments $M_{AB}$ and $M_{BA}$, as well as the distortion moments for the other members of the frame, is indicated by the dotted lines in Fig. 31(*e*). The final moments are indicated by full lines. The moment diagram for member AB, showing also the direction of the moments and forces, is presented in Fig. 31(*c*) in a convenient form for memorizing.

By statics:

$$-\,2{,}812\,k = 200 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

in which $k = -\,0.071$. Eq. 87 is similar in form to Eq. 25. The final moments are equal to the algebraic sum of the moments in Fig. 31(*a*) and Fig. 31(*e*), multiplied by factor $k = -\,0.071$. As shown graphically in Fig. 31(*f*), these final moments differ slightly from those computed by the author.

The drawing can be included on a sheet the size of an ordinary business letter, although greater accuracy, where required, may be obtained by using a larger scale. The sidesway of members with haunches may be determined similarly, necessary constants being computed independently, or scaled (23) from diagrams.

To summarize: The author declared the purpose of his paper (see heading, "Synopsis") to be "to demonstrate the usefulness of the principle of superposition in the analysis of rigid frames by the slope-deflection and moment distribution methods." Granting the usefulness of the principle of superposition, it is the latter part of the foregoing statement which the writer questions. The writer's purpose has been to plead the virtue of more simple design procedures in demonstrating the acknowledged worth of the principle of superposition.

VICTOR R. BERGMAN,[15] ASSOC. M. AM. SOC. C. E.[15a]—For the analysis of those rigid frames in which the effect of joint translation as well as of joint rotation must be considered, Professor Wilson has presented a useful, interesting method of procedure involving superposition. Although some of the problems in this paper can be solved more readily by other approaches, all the examples have a definite value in developing and illustrating the method.

[15] Structural Designer, York and Sawyer, Architects, New York, N. Y.

[15a] Received by the Secretary October 2, 1944.

One of the frames analyzed by Professor Wilson, the gable bent in Fig. 10, has enjoyed quite a vogue in technical literature. The author's treatment of this problem is instructive, and indicates the ease of application of the method to this type of structure, when the component members are of constant cross section. However, the growing demand for artistically pleasing structures leads to the construction of many of these frames with members of varying cross section. In this case, a gable bent often can be analyzed more advantageously by some other method such as column analogy.

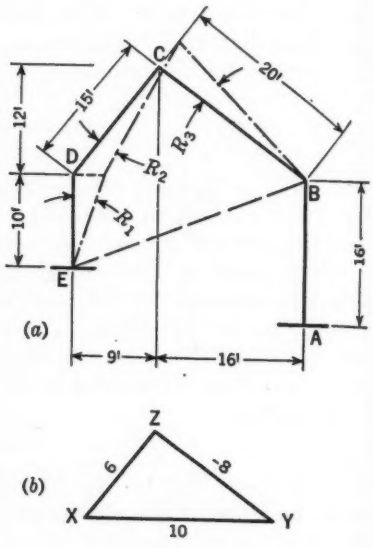

FIG. 32

It may be of value to point out that the "Relation Between Values of $R$ for Members of a Quadrangle" (see subheading under "Nonrectangular Frames") can be determined readily by a semigraphical method. The gable bent in Fig. 10, redrawn in Fig. 32(*a*), will be used for purposes of illustration.

In Fig. 32(*b*), using any convenient scale, lay off a length, XY, of 10 units at right angles to leg DE of the gable bent. Through points X and Y draw lines perpendicular, respectively, to members CB and CD, intersecting at point Z. The values of XZ and YZ are found, by scaling, to be 6 and 8 units, respectively. Then: $R_1 = \frac{10}{10} R_1 = R_1$; $R_2 = \frac{8}{15} R_1$; and $R_3 = \frac{6}{20} R_1 = \frac{3}{10} R_1$.

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(23) **"Analysis of Continuous Frames by Graphical Distribution of Moments,"** by A. A. Eremin, published by the author.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# PRINCIPLES OF DEPRECIATION

## REPORT OF THE SPECIAL COMMITTEE AUTHORIZED BY THE BOARD OF DIRECTION TO ANALYZE AND DISCUSS THE 1943 REPORT OF THE NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS' COMMITTEE ON DEPRECIATION

**Discussion**

BY R. C. PRICE

R. C. PRICE,[29] ASSOC. M. AM. SOC. C. E.—The following comments are limited to a discussion of the divergencies of view expressed in the Society committee's report. Emphasis is laid on the regulatory application of depreciation excluding consideration of depreciation for commercial valuation and other purposes. The Society committee is to be congratulated for giving attention to this important and controversial matter.

Any one venturing in the field of depreciation is impressed with the lack of consistency in the nomenclature. The late Justice Brandeis once said that "Value is a word of many meanings," and the same has been true and is true of the word "depreciation." In the light of the recent wise action by the U. S. Supreme Court which has had the effect of bringing some order out of the former confusion in regulation terms and the move of NARUC to bring its concepts in line, it behooves all those who are interested as technicians to refrain from clinging wistfully to the nebulous idea of "value" as expressed in the old Smyth versus Ames decision and instead to accept gratefully the shift from an impracticable concept of "fair value" to a useful and realistic "cost" basis. Such acceptance would result in a substantial revision of the Society report.

*Definition of "Depreciation."*—The Society committee favors the definition of "depreciation" given in the Lindheimer Case and that from the existing

NOTE.—This report was published in June, 1944, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: September, 1944, by C. Beverley Benson, E. M. T. Ryder, Wallace B. Carr, J. L. Campbell, Paul L. Holland, J. Kappeyne, C. Terry duRell, Anson Marston, Terrell Bartlett, E. E. Hart, and Luther R. Nash; and October, 1944, by Donald Gunn.

[29] Asst. Prof., College of Eng., New York Univ., New York, N. Y.

NARUC system of accounts. Both of these relate to value under the old regulatory concept. The definitions favored by the Society committee however are inconsistent with what has been sanctioned by the Supreme Court in the Hope Natural Gas Case. The impracticable concept of "fair value" has been pushed into the background and "original cost" has been given the star rôle. The "loss in service value" which the Society committee evidently favors has been superseded by the shift in emphasis from value to cost. Consequently, one need not expect the "remaining service value" (which borders on the indeterminate, at best) to be synonymous with the "remaining service life." The question then is whether the "remaining service life" based on accounting facts is a reasonable basis for handling depreciation.

*The Concept of Depreciation.*—In its discussion of the depreciation concept the Society committee apparently fails to recognize that utility accounts also reflect not value but cost. Under the concept of the Society committee a steam turbogenerator built in the 1930's might thus far have escaped depreciation because its value, in terms of current price levels or of capitalized net income, would have appreciated. Depreciation allowance in this case is essential as a means of allocating the cost, not the value, of the generator over its useful life. By the same token there is reason to question the Society committee's view that the "loss of value at any date should be the basis of all accounting and financial policies with reference to depreciation."

The depreciation concept which is applicable to regulated utilities is the one that J. C. Bonbright[30] has identified as "amortized cost." Under that concept (admittedly an accounting concept), it appears reasonable and correct to regard accrued depreciation as equivalent to the depreciation reserve, the written-off part of the plant which is still in use. It must be confessed that, although this point was before the Supreme Court in the Lindheimer Case, there has not been a clear expression of Court opinion. (In that case, however, the Court did recognize the "propriety of basing annual depreciation on cost.")

*Straight-Line Depreciation.*—The Society committee objects to the NARUC report's indicated preference for straight-line depreciation accounting and "emphasizes that actual depreciation is factual and that accounting, to be correct, must reflect the facts of actual depreciation." If the implication is that the Society committee favors reliance on annual estimates of depreciation, rather than on any systematic provision for depreciation, that alternative is subject to certain criticism and difficulty. For one thing the amount of depreciation is thrown into a zone of uncertainty since the opinion of one expert will differ from that of another expert. In addition the time required for determinations may virtually defeat the purpose. There is a clear gain in making systematic provision for depreciation through either straight-line or annuity method.

The Society committee is well advised in questioning the NARUC position that actual depreciation (whatever that is) conforms to the depreciation reserve accrued by the straight-line method. One may observe that it requires some temerity to assert that either the annuity or the straight-line method will

[30] "Valuation of Property" by J. C. Bonbright, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, Vol. I, pp. 185–187.

produce close agreement with "actual depreciation" or what the Society committee calls "loss in service value." Fortunately, exactitude is not essential here and it is not of prime importance that neither method produces close conformity.

In the writer's opinion the straight-line method is superior to the annuity method for purposes of regulation because:

(1) It is in wider use;
(2) It is simpler in application;
(3) It provides a larger early reserve against earlier-than-expected retirement and has been said to result more probably than not "in an under-allowance" whereas the annuity method would even more frequently result in under-allowance; and
(4) It is unaffected by fluctuating levels of "reasonable rate of return" unlike the annuity method reserve.

The broader use of the straight-line method is attested by a check of methods used by 460 large industrial corporations as reported by Solomon Fabricant.[31] Of the 460 corporations "206 stated explicitly that they used the straight-line formula" and "another 226 companies implied its use in their statements." The Interstate Commerce Commission emphasizes the straight-line method for utilities under its jurisdiction.

The greater simplicity of the straight-line method is apparent on examination. Using that method the depreciation charge is computed by simple arithmetic. The literature shows that confusion in using the annuity method has sometimes led to mistaking the uniform annual sinking-fund deposit for the depreciation charge. In addition, the interest-table applications used in the annuity method are decidedly more complicated than the simple calculation for the straight-line method.

In a symposium on the "Fundamental Aspects of the Depreciation Problem," J. A. Walls, M. Am. Soc. C. E., stated,[32] "Such estimates [of future life for most electric utility property], even for the near future, have limited accuracy and reliability, and, when extended into the more distant future, they would appear to have even less accuracy and reliability." If this uncertainty is general, it is only just to the investors to build up a reasonably large early reserve to guard against premature retirement for any reason. It is commonly recognized that the reserve for the straight-line method consistently exceeds that for the annuity method except for the terminal year of estimated life.

The NARUC report urges (see heading, "H. Depreciation in Rate Making") that when the sinking-fund method is used in rate proceedings, "the interest rate employed should be the same as the rate of return which is applied to the undepreciated rate base." Whether the recent downward trend of interest rates continues or not, as the rates of return which the courts feel to be reasonable for given utilities change from time to time, there will be a complicating effect on the depreciation reserve geared to the annuity method, whereas the straight-line reserve is independent of the rate of return.

[31] "Capital Consumption and Adjustment," by Solomon Fabricant, *Publication No. 35*, National Bureau of Economic Research, New York, N. Y., 1938, p. 65.

[32] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1240.

No one should admit more quickly than engineers the need for bench marks to make regulation work. The new reliance on cost rather than value gives one needed bench mark. A second needed benchmark is achieved by tying the deduction for depreciation to the depreciation reserve; and use of the straight-line method rather than the annuity method will provide the third. Under the stimulus of recent Supreme Court cases, valuation engineers have been encouraged to go "half a hog" in reducing the uncertainties surrounding "value" by substituting "cost"; now they should go "the whole hog" in taking the results out of the realm of conjecture and placing the rate-setting procedure on the solid ground of prudent investment, less amortized cost calculated on a straight-line basis.

*Retroactive Adjustment of the Depreciation Reserve.*—This presents the most ticklish problem in the depreciation picture. According to the Society's report, the NARUC report recommends

> "* * * that where the reserve computed by the straight-line method substantially exceeds the reserve currently in the property that the adjustment necessary to bring the current reserve up to the depreciation reserve computed on the straight-line basis should be through charge to surplus or special section of the income account."

The NARUC report also recommends that "the objective of correcting inadequate reserves should be approached with appropriate consideration of the practical effects of alternative courses of action." Granted the ambiguity of the NARUC report in this respect, the question is whether a withdrawal from surplus should be made to bring current reserves up the reserve calculated on the straight-line basis, or whether the remaining net cost should be spread over the remaining lives of properties by the straight-line method. Retroactive adjustment should certainly be avoided where utilities in good faith have followed a sinking-fund depreciation policy. Only in exceptional cases of bad faith should it become necessary to charge to surplus instead of spreading the remaining net cost of the property over remaining lives. The unreasonableness of changing the rules in the middle of the game is so apparent that the Society committee is well warranted in protesting the offending clause. However, the writer does not believe that it was the intention of the NARUC committee to require general retroactive adjustment, and the NARUC committee would probably agree with the point made by L. R. Howson,[33] M. Am. Soc. C. E., before the American Water Works Association that water utilities as a whole cannot and should not be required to charge surplus to make up a difference between existing reserves and what the reserves would be under the straight-line method. A statement by A. R. Colbert, a member of the NARUC committee, is informative.[34] He wrote: "* * * any general requirement for retroactive application of depreciation is unnecessary and should not be required. This is the plain import of the committee's report," and again, "* * * no requirement for general retroactive depreciation accounting is recommended by the committee."

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[33] See *Engineering News-Record*, July 13, 1944, p. 94.

[34] *Journal of Land and Public Utility Economics*, May, 1944, p. 94.

It may be that retroactive adjustment should be required where there is a record both of previous bad faith and also of a substantial relative difference between annual depreciation charges calculated on a straight-line basis throughout and annual charges calculated by spreading the remaining net cost over remaining lives. This latter contrasts with the criterion suggested in the NARUC report of the "difference between the book reserve and the proper (straight-line) reserve." In addition it is not clear whether the "difference" referred to in the NARUC report is to be measured absolutely or relatively.

*Summary.*—The Society report seems to err in clinging to the old "fair value" concept instead of embracing the "original cost" concept which bulks increasingly larger in the regulatory framework. It should be recognized that accounts also properly reflect cost rather than value. The importance of adopting workable and practical procedures for handling utility regulation deserves more emphasis. The need is for streamlining regulation by associating it with recognizable bench marks so that regulation may be more useful and effective in producing satisfactory service at reasonable rates and so that the investments of individuals and companies may be better protected.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# MILITARY AIRFIELDS
# A SYMPOSIUM

## Discussion

BY IRA B. MULLIS, AND C. F. IZZARD

IRA B. MULLIS,[31] ESQ.[31a]—This Symposium is one that deserves to be widely read and discussed. Apparently Brigadier-General Stratton has found that the AASHO Method (55)[31b] is an inadequate one; otherwise, he would not have modified the method by utilizing a 10-lb rammer dropped from a height of 18 in. in five sets of twenty-five drops each (see heading, "Compaction Tests and Control"). In terms of compactive work applied, this is more than four times that of AASHO Method.

The writer has also found that the AASHO Method (55) is entirely inadequate, except in cases of well-graded sandy clay or other materials of a similar nature. Many failures over a widespread area have been observed where this standard of compaction was followed. Flexible pavements have failed in many cases under rolling operations during construction or later. Of course, the roadbed at the time of failure was wet enough to be plastic. To complete the work over wet spots that had failed, the pavement was removed and later patched when dry. Since patching does not alter the porous condition of the roadbed, other failures follow during wet periods. One project is recalled in which a concrete pavement, built under the general supervision of an ardent advocate of the AASHO Method (55), had settled and broken so that much of it has been raised by means of the mud pump to overcome the depressions.

Although General Stratton states that the service performance of airport pavements under severe operating conditions has been extremely gratifying, he makes no prediction as to final results to be expected except to state that the

NOTE.—This Symposium was published in January, 1944, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by James B. Newman, Jr.; March, 1944, by Thomas E. Stanton; April, 1944, by W. E. Howland, and David S. Jenkins; May, 1944, by William E. Rudolph, Raymond L. Irwin, G. G. Greulich, Hibbert Hill, Jacob Feld, and Robert Horonjeff; June, 1944, by Clarence S. Jarvis, D. D. McGuire, E. D. Parmer, A. Casagrande, and Harry E. Cotton; and September, 1944, by D. O. Murray, Thomas B. Pringle, John R. Haswell, W. J. Turnbull and William H. Jervis, Ralph R. Proctor, D. M. Burmister, Ralph Hansen, H. V. Pittman, G. R. Schneider, and H. M. Williams.

[31] Research Engr., Southern Aggregates Corp., Raleigh, N. C.

[31a] Received by the Secretary October 2, 1944.

[31b] Numerals in parentheses, thus: (55), refer to corresponding items in the Bibliography at the end of the Symposium, and at the end of discussion in this issue.

optimum moisture condition at which the subgrade and base courses are placed does not represent the moisture condition after the passage of time. Data collected by Bert Myers (56) tend to support this statement.

Apparently the water content at the time of compaction has no effect on controlling the future water content, though pore space does have a marked effect as shown by Mr. Myers' data (56*a*). The writer has many data (a considerable part of which has not been published) in support of the statement that limiting the pore space is a means for controlling the water content (57). One reason for the strong resistance of hard pan and consolidated sandy clay to loads and to the penetration of water is its very limited pore space.

Although many hold that certain static or impact forces can be used as standards for forecasting the ability of roadbed materials to resist traffic and weather, the writer knows no data that will support this belief. Indeed, if one should supplement pressure with high-frequency vibration, the method would not constitute an adequate standard of stability, although it might represent the maximum compaction obtainable by any method. Furthermore, work is not a measure of results to be expected; but skill in applying work, and the ease with which certain types of materials are formed, are items that must be considered. In view of this fact, the result obtained in making a roadbed resistant to traffic and water must be the sole standard of quality.

Fig. 41 is a well-suited illustration of the relations of density, water content, voids, and resistance to force. It shows that resistance to force or penetration decreases uniformly with increase in water content from 2,500 lb to about 1,000 lb at which point resistance to force decreases rapidly. Above the 1,000-lb point the curve shows that the body of soil is rigid, but below this point the soil is plastic. The proportional limit having been passed, near the 1,000-lb point, none of the loads corresponding to the densities and moisture contents of about 121.5 lb per cu ft and 13%, respectively, can be considered safe for the load imposed unless confined.

If the moisture-density curve and the moisture-penetration curve were extended to the left as far as they are to the right, the moisture-penetration curve would begin to drop at a point in excess of 2,500 lb. The cause of the drop of the curve at this point would be due to an increase of air content. As the density of the mass and its water content are lowered, the air content increases and thereby changes from a state of rigidity at 2,500 lb to a state of plasticity due to excessive volumes of air.

If the dry weight curve were extended downward to the left to a weight of 100 lb, the voids would be 39.6% on the basis of an assumed specific gravity of 2.65; at 90 lb the voids would be 45.7%; and at 80 lb, 51.7% with little resistance to loads or the penetration of water. Consequently, the penetration resistance curve on the dry side (left) would decrease in a manner similar to the wet side of the curve.

Although engineers are aware of the softening effect of water on roadbeds, few seem to realize that rutting or plastic flow does not start until a certain amount of water is absorbed. Of course a small amount of water in pore spaces is desirable at all times, but when a critical amount is absorbed the material is plastic, as shown by Fig. 41 where a less than 2% increase of water changes the

resistance from 850 lb to 475 lb. Fig. 42 shows a similar critical change near the 750-lb point.

Elsewhere, the writer (58) has presented many data concerning this critical change, which he had previously indicated in 1938 (57). Inasmuch as the presence of a critical amount of water in a clay body produces a sharp and definite change in its structural properties it conforms to Hooke's law and must be so regarded. On the one hand, a consolidated body of dry or moist clay that is overstressed fails by rupturing, but one that is critically wet fails through plastic distortion. That which marks the boundary between the rigid and plastic states is rupture, a property universally recognized by physicists. It would seem, therefore, that bodies of earthy sediment containing critical amounts of air or water, or both, should be expected to fail even under light loads.

The method of guarding against this form of weakness is discussed elsewhere (57)(58).

C. F. IZZARD,[32] ASSOC. M. AM. SOC. C. E.[32a]—A comparison of actual runoff hydrographs resulting from simulated rainfall with theoretical hydrographs computed by Eq. 5 for two types of surface—smooth pavement and dense turf—is contained in this discussion. The supporting data are taken from a cooperative project for which the Soil Conservation Service of the U. S. Department of Agriculture performed the experiments and the Public Roads Administration of the Federal Works Agency directed the project and analyzed the results. The equipment used and the method of performing tests are described in detail elsewhere (59). Fig. 49 shows the equipment in operation.

The "paved surface" was actually an asphaltic roofing paper with crushed slate particles on the surface, laid on a wooden platform covered with an asphaltic mastic. The surface conformed closely to a true plane, probably more accurately than the average surfaced runway. The turf surface was obtained by placing a dense bluegrass sod upon the paved surface. The soil was about 1.5 in. thick. Movement of water beneath the soil was checked by transverse wooden strips. Tests were made with the soil in a saturated condition so that infiltration was almost negligible.

A typical hydrograph with paved surface on a 1% grade is shown as a solid line in Fig. 50. The rain began at zero minutes and continued beyond the time at which equilibrium was reached. The dashed line shows the theoretical hydrograph computed by Eq. 5 for the indicated rainfall intensity, length of overland flow, and slope, with $n = 0.02$ as recommended by Mr. Hathaway for smooth pavement. The dotted line shows the best fit obtainable with Eq. 5 by shifting the time axis and adjusting the value of $n$ so as to fit the slope of the hydrograph at one third the maximum runoff intensity, which is the point of inflection on the theoretical curve. The time shift in this case was 0.52 min as shown in Fig. 50, and the adjusted value of $n$ was 0.00603.

The average percentage deviation of the unadjusted computed hydrograph from the actual hydrograph was 18%, based on ordinates at 1-min intervals

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[32a] Received by the Secretary October 5, 1944.

excluding the first minute. The average percentage deviation of the adjusted computed hydrograph from the actual hydrograph was only 1%.

Similar comparisons were made for typical runs at 3.6 in. per hr and 1.8 in. per hr on lengths of 12 ft, 24 ft, and 48 ft, and for 72-ft plots on slopes of 2%

FIG. 49.—RAINFALL SIMULATOR IN OPERATION ON BLUEGRASS TURF PLOT, 72 FT LONG; RAINFALL INTENSITY, 3.81 IN. PER HR; SLOPE OF PLOT, 1%; RUNOFF RATE APPROACHING EQUILIBRIUM

and 4%. Results were similar to those in Fig. 50, although the adjusted computed curve shown in this figure fitted better than any of the others. In all cases it was necessary to shift the computed hydrograph on the time axis and to change the value of $n$ from the recommended value of 0.02 in order to obtain a reasonably good fit. For unadjusted computed hydrographs on 1% slope ($n = 0.02$) the average deviation was about 15%, generally decreasing with decrease in the length of plot. The deviation tended to increase as the slope of the plot increased. The computed hydrographs, adjusted and unadjusted, were always at or below the actual hydrograph, except during the first minute.

The adjusted computed hydrographs on the 1% slope averaged about 5% deviation from the actual hydrograph. The deviation appeared greatest for the 24-ft and 48-ft plots, and increased as the slope of the plot increased. The adjusted values of $n$ varied from 0.005 to 0.010. The shift in the time axis to bring the adjusted hydrograph into position with the actual hydrograph at the one-third point ranged from 0.3 min to 0.8 min.

The average percentage deviation for both adjusted and unadjusted computed hydrographs as compared to the actual hydrograph was generally greater for the 1.8 in. per hr rainfall intensity than for the 3.6 in. per hr rainfall intensity.

For the turf plot the computed hydrograph shown in Fig. 51, based on the recommended value of $n = 0.80$ for dense turf at a rainfall rate of 3.81 in. per hr, deviated from the actual hydrograph by about 16%. The adjusted computed hydrograph deviated about 6%. The latter deviation was about the same for the other lengths of plot at the same rainfall intensity and occurred principally in the upper half of the hydrograph. The tendency for the actual hydrograph to differ from the theoretical curve was common to all turf runs and was observed in runs made under actual field conditions.

The unadjusted computed hydrograph for the 1.83 in. per hr rate of rainfall, on a 72-ft plot sloping 1%, deviated 39% from the actual hydrograph. The adjusted computed hydrograph for this case, with $n = 0.429$ and a time shift to the right of 1.33 min, deviated less than 2% on the average from the actual hydrograph.

The data so far available do not justify changes in the values of $n$ recommended by Mr. Hathaway or the introduction of an arbitrary time displacement.

The tests here reported did not extend beyond a 3.8 in. per hr rainfall intensity on a 72-ft plot. Within this range the evidence is fairly conclusive (60) that discharge on both paved and turf surfaces is proportional to the third power of the depth instead of the second power, as indicated by the exponent $M = 2$ in Eq. 5. When one attempts to use $M = 3$ in Eq. 4, the turbulence factor $I$ becomes zero, and the factor $K$ in Eq. 3 becomes infinite. Furthermore, the general formula, Eq. 2, was derived by Mr. Horton using an assumed value of $M = 2$ for which the integration of his differential equation resulted in

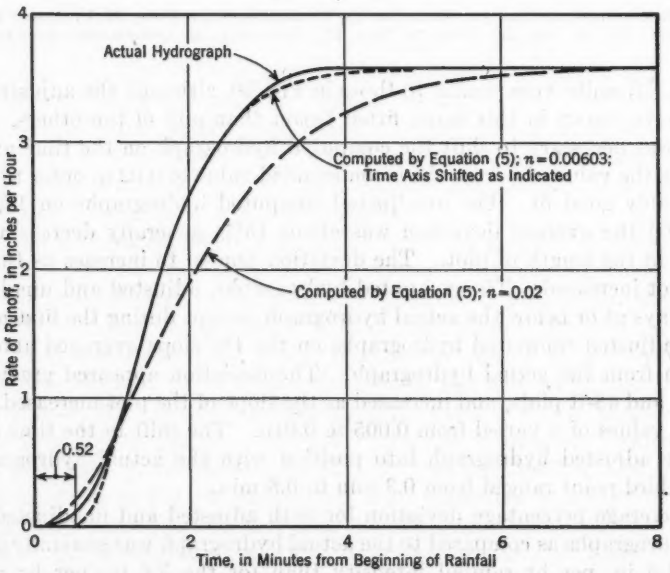


FIG. 50.—ACTUAL AND COMPUTED HYDROGRAPHS FOR PAVED SURFACE (LENGTH OF PLOT, 72 FT; SLOPE, 1%; AND RATE OF RAINFALL, 3.57 IN. PER HR)

the hyperbolic tangent function. For $M = 3$, integration results in a more complex function; and for other than whole-number values of $M$, integration becomes almost impossible. Mr. Horton states (18) that the equation here referred to as Eq. 2 is only quasi-rational for values of $M$ other than 2.

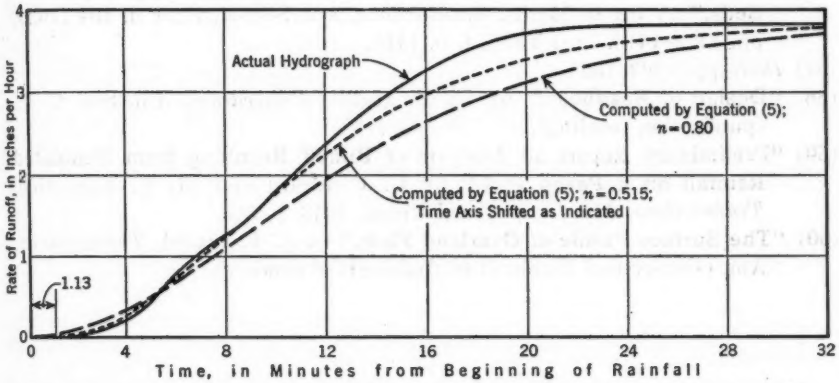


FIG. 51.—ACTUAL AND COMPUTED HYDROGRAPHS FOR TURF SURFACE (LENGTH OF PLOT, 72 FT; SLOPE, 1%; AND RATE OF RAINFALL, 3.81 IN. PER HR)

It is doubtful that the viscous flow condition, which apparently exists within the range covered by the tests reported herein, continues to exist for greater rates of discharge. A limited series of tests performed on the paved surface by admitting water at the upper end only (60) indicates that, for overland flow undisturbed by rainfall, the viscous type of flow terminates at about $\mathbf{R} = 2{,}000$ (Reynolds' number) corresponding roughly to runoff at 4 in. per hr from a 72-ft plot. Beyond that range the data are inconclusive and incomplete; but, since flow in pipes becomes fully turbulent beyond $\mathbf{R} = 2{,}500 \pm$, it appears reasonable to assume that overland flow would also become fully turbulent in the same range.

Consequently, from a scientific viewpoint, it appears likely that, with rainfall at 4 in. per hr, discharge is proportional to the cube of the depth up to length of overland flow approaching about 80 ft, and becomes turbulent, with discharge proportional to depth to the five-thirds power for greater lengths. Further experiments are needed to establish what actually happens.

The concept of the factor of turbulence $I$, as used in Eq. 4, appears open to question although it is probable that Eq. 5 gives a rough approximation of actual hydrographs on lengths of overland flow exceeding 100 ft. There is some doubt in the writer's mind as to whether elaborate developments based on this equation are warranted, although it must be admitted that nothing better exists at the present time.

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(18) **"The Interpretation and Application of Runoff Plat Experiments with Reference to Soil Erosion Problems,"** by Robert E. Horton, *Proceedings*, Soil Science Soc. of America, Vol. 3, 1938, pp. 340–349.

(55) **"Standard Laboratory Method of Test for the Compaction and Density of Soil,"** A.A.S.H.O. Designation: T 99–38, *Standard Specifications for Highway Materials and Methods of Sampling and Testing*, A.A.S.H.O. Pt. II, 1942.

(56) Discussion by Bert Myers on **"Principles Applying to Highway Road-Beds,"** by Ira B. Mullis, *Transactions*, Am. Soc. C. E., Vol. 104 (1939) pp. 1412–1416. (*a*) Table 4, p. 1413.

(57) *Ibid.*, pp. 1393–1394.

(58) **"Design of Roadbeds,"** by Ira B. Mullis, *Proceedings*, Am. Soc. C. E. (publication pending).

(59) **"Preliminary Report on Analysis of Runoff Resulting from Simulated Rainfall on a Paved Plot,"** by C. F. Izzard and M. T. Augustine, *Transactions*, Am. Geophysical Union, 1943, p. 500.

(60) **"The Surface Profile of Overland Flow,"** by C. F. Izzard, *Transactions*, Am. Geophysical Union, 1944 (publication pending).

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# FLOOD FORMULAS BASED ON DRAINAGE BASIN CHARACTERISTICS

## Discussion

BY C. O. CLARK, JUN. AM. SOC. C. E.

C. O. CLARK,[41] JUN. AM. SOC. C. E.[41a]—The expression of flood formulas in terms of drainage basin characteristics—a more comprehensive classification than drainage area—has been needed for many years. The authors' proposal that the unit hydrograph be used as a basis for the expression of flood experience is the soundest procedure yet available.

The authors' emphasis on the element of frequency is important. The contemplation of maximum possible floods is appropriate in extending the vision of planners to the magnitude of possible floods; but to regard such floods as truly the maximum possible is to ignore their derivation from actual rainfall and climatological data. It seems probable that, as more records of phenomenal rainfall are collected, estimates of maximum possible rainfall will be revised upward. Excellent as it is, the technique of estimating maximum possible rainfall is an infant whose horizon will be extended as it matures.

As the understanding of the possible magnitude increases, the impracticability of structural provision for it will become increasingly apparent, but the alternative of economic protection by insurance will remain, and more planners will turn to this alternative. With increasing demand for flood insurance will come a greater demand for flood formulas which express the probability of floods of various magnitude.

In applying the formulas in the paper to streams of unusual shape in Virginia and North Carolina, the writer found better correlation with the formulas for runoff depth (Eqs. 3, 4, and 5), than with the flood flow formulas (Eqs. 6*a*, 6*b*, and 6*c*). Some of the universal quality which the authors incorporated into their data by the use of unit hydrographs and runoff depths may have been lost in the later attempt to express results in terms of so few

NOTE.—This paper by H. B. Kinnison, M. Am. Soc. C. E., and B. R. Colby, Esq., was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1944, by Clarence S. Jarvis; September, 1944, by L. R. Beard, and M. J. Ord; and October, 1944, by Howard M. Turner, and Byron O. McCoy.

[41] Hydr. Engr., U. S. Engr. Office, Winchester, Va.

[41a] Received by the Secretary October 4, 1944.

variables. The data as presented may be wholly adequate for the expression of New England flood experience. The following comments are intended to suggest a technique by which the data could be presented in a manner more easily applicable to wider geographic areas, and by which some of the influences which the authors examined but were unable to analyze possibly could be expressed.

The authors' presentation of flood formulas in terms of drainage basin characteristics and probability (or frequency) of occurrence involves combining several logical elements in the analysis. Some of these elements are much stronger than others; one appears weak. Perhaps separation of the elements will allow a stronger recombination with less dependence on the weaker elements. Basically, the logical elements appear to be:

(*a*) The expression of relative flood size for various frequencies in terms of the frequency of runoff volume—

1. Assuming runoff to vary in proportion to the combined flood experience of some streams;
2. Assuming frequency of recorded point rainfall to be indicative of frequency of flood-producing rainfall on an area and indicative of runoff volume frequency;
3. Assuming a constant chronological sequence of this rainfall or runoff to be equally critical on all watersheds;
4. Assuming runoff volume to be uniformly distributed over the watershed; and
5. Assuming variation of the runoff volume with watershed characteristics.

(*b*) The expression of the flood characteristically produced by a watershed from a given amount of runoff, in the unit hydrograph, assuming that this element is not constant but that the peak flow per unit volume increases with total runoff volume.

(*c*) The expression of the flood peaks of probable expectancy as the product of elements (*a*) and (*b*).

(*d*) The expression of element (*c*) as a flood discharge formula in terms of characteristics of drainage area.

The writer is in accord with the basic elements (*a*), (*b*), and (*c*). Possibly these steps could be strengthened if dependence on some of their subelements could be increased and extended and that on others reduced. Thus, element (*a*)1 has a basis in provable flood experience which makes it a very strong factor, whereas element (*a*)5 is likely to be obscured by many other influences and to require considerable data to prove its nature conclusively. The writer proposes to extend these logical uses by discussion and examples.

Element (*d*) tries to correlate variation in element (*c*) with watershed characteristics. This attempt can be no more successful than an expression of the unit hydrograph in terms of watershed characteristics. The writer's attempts at the latter have been only moderately useful when utilizing a

graphical expression of watershed shape and depressingly inadequate if confined to terms of slope, watershed area, and watershed length—the authors' selected variables. The authors have clearly indicated the difficulties of simple correlation of flood peaks and watershed characteristics.

The writer proposes to abandon element (*d*) and to express element (*c*) as a multiple of the peak of the unit hydrograph, leaving the user free to adapt the unit hydrograph of his particular stream. This proposal is based upon the writer's experience that unit hydrographs are secured rather easily from records if time and cost permit; that, even when developed from short records, such unit hydrographs are more consistently reliable than if developed by any method of synthesis known to the writer; and that, if the flood producing characteristic of a watershed must be expressed from topographic maps, there are means available which exceed in accuracy any expression possible in terms only of *s*, *M*, and *L*. However, it appears that the authors have collected considerable data on watershed characteristics and unit hydrographs. Direct correlation of unit hydrograph peak with watershed characteristics may be possible. Such correlation is highly desirable, and, if possible, it would help to solve both this problem and many related problems depending on synthesis of the unit hydrograph.

The proposed modified expression would parallel the foregoing logical theory, but would place slightly different emphasis on some elements, thus:

(*A*) Relative flood size for various frequencies would be expressed in terms of frequencies of runoff volumes equal, or equivalent in peak flow produced, to multiples of a uniformly distributed unit volume generated in unit time:

1. Although this definition does not exclude the use of the applicable assumptions of element (*a*) in the authors' procedure, it by-passes numerous controversies regarding the absolute truth of these assumptions by including equivalents as well as equalities.
2. There is an opportunity to include the authors' variation of peak of unit hydrograph as the equivalent of a larger volume of runoff, and thereby to avoid the controversies between those who attribute the variation in actual flood peaks to variation in the unit hydrograph and those who attribute it to intensification and concentration of runoff volume as compared to the assumption of uniformity in unit hydrograph theory. Regardless of the merit of either viewpoint, a distinction is not essential to the problem at hand.

(*B*) The flood peak characteristically produced by a watershed would be that resulting from uniformly distributed unit volume generated in unit time.

(*C*) This would be the same as element (*c*) in the authors' procedure.

In deriving element (*A*), the authors' use of combined actual stream-flow records may be extended to include more records of similar streams, because the peak flows may be known, even though the records may not be complete enough to define the actual flood volume. The question of similarity may be perplexing, and any one can propound elements of hypothetical dissimilarity, whose influence may be easily exaggerated. The writer's technique is to assume simplicity and similarity until the preponderance of evidence indicates

otherwise. Thus, it may be desirable to combine all or most data available as a single record, assuming similarity, and later to analyze smaller groups of data of more definite similarity in a particular characteristic to ascertain if that particular characteristic is significant. Thus, a large group of streams all sheltered by mountain ranges over 2,500 ft in elevation might be found to experience records 25% lower than the mean of all data. However, it takes many records to prove this kind of variation conclusively. It must be remembered that the casting of three 7's in succession is not proof that dice are loaded.

In illustration of the simplified technique, an analysis of 834 flood occurrences is presented. The aggregate record totals 734 years, collected at 56 recording stream gaging stations in the adjacent watersheds of the Potomac, James, and Roanoke rivers. The stations are listed in Table 4. All these major watersheds drain some part of Virginia east of the Allegheny Mountains. The largest stream included is the Potomac River at Point of Rocks, Md., with a watershed of 9,651 sq miles, and the smallest is Georges Creek at Franklin, Md., with a watershed of 72 sq miles. This very large range in drainage area has been included in one analysis to bring out the possibilities of this form of analysis. The group of data is then subdivided into lesser groups of more similar drainage area in an attempt to determine whether drainage area alone is a significant variable affecting relative frequency of floods.

In each analysis all peak flood discharge occurrences have been listed in order of the magnitude of the ratio by which they exceed the peak flow produced by 1.0 in. of runoff uniformly generated over the drainage area at a uniform rate during a period of 6 hours. These 834 flood ratios in descending order have been treated as a single record of 734 years in the determination of the frequency curve shown in Fig. 19(*a*).

The use of the frequency curve shown to determine flood discharge frequency on any particular small stream in the same region would involve several assumptions, namely:

I. That the frequency with which the unit flood discharge is exceeded on small watersheds is the same as that for the average of all watershed sizes in the region;

II. That the effects of other drainage area characteristics are the same as for the averages of all watersheds; and

III. That, in the region, the peak of the unit hydrograph of a single stream can be determined with suitable accuracy.

One should maintain an open mind regarding the accuracy of an hypothesis, such as assumption I, and demonstrate its correctness when the sample of data becomes large enough to be conclusive. The writer has never worked with a single flood record long enough to support a conclusive judgment. However, analysis of smaller groups of aggregate records from more assuredly similar watersheds may be qualitatively indicative. Size of drainage area alone was studied in five analyses, the results of which are represented in Fig. 19(*b*). In these analyses, the aforementioned record of 834 occurrences was divided into groups of more nearly similar watershed area as indicated in Fig. 19(*b*).

TABLE 4.—FLOODS OF THE MIDDLE ATLANTIC SLOPE RIVERS

| Stream | Watershed | | | | Unit flood peak[c] (cu ft per sec) | Flood Record | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Station | Drainage area (sq miles) | Length[a] (miles) | Fall[b] (ft per mile) | | Years | Flood Ratios[d] | | |
| | | | | | | | Largest | Number[e] | Number per year |
| Shenandoah River | Millville, W. Va. | 3,040 | 197 | 5.8 | 37,800 | 28.8 | 6.09 | 23 | 0.80 |
| Meherrin River | Lawrenceville, Va. | 553 | 67 | 3.2 | 6,700 | 11.5 | 5.66 | 14 | 1.22 |
| Cacapon River | Great Cacapon, W. Va. | 677 | 107 | 9.5 | 15,600 | 9.0 | 5.62 | 4 | 0.44 |
| Potomac River | Hancock, Md. | 4,073 | 179 | 6.5 | 61,000 | 19.3 | 5.57 | 16 | 0.83 |
| Roanoke River | Weldon, N. C. | 8,445 | 280 | 3.9 | 48,200 | 41.8 | 5.41 | 151 | 3.62 |
| Goose Creek | Leesburg, Va. | 338 | 33 | 11 | 8,330 | 3.9 | 5.38 | 1 | 0.26 |
| Rivanna River | Palmyra, Va. | 675 | 50 | 8.0 | 14,700 | 9.1 | 5.15 | 10 | 1.10 |
| South Branch, Potomac River | Springfield, W. Va. | 1,471 | 120 | 11 | 28,400 | 14.2 | 5.03 | 10 | 0.71 |
| Willis River | Flanagans Mills, Va. | 247 | 54 | 4.2 | 2,300 | 6.5 | 4.87 | 12 | 1.86 |
| Potomac River | Shepherdstown, W. Va. | 5,936 | 234 | 4.6 | 70,000 | 15.1 | 4.78 | 15 | 1.00 |
| Banister River | Halifax, Va. | 552 | 46 | 7.2 | 5,500 | 14.4 | 4.73 | 17 | 1.18 |
| North Fork, Shenandoah River | Strasburg, Va. | 772 | 96 | 12 | 21,700 | 12.1 | 4.60 | 5 | 0.41 |
| Potomac River | Point of Rocks, Md. | 9,651 | 257 | 4.4 | 105,000 | 48.3 | 4.54 | 40 | 0.83 |
| North Branch, Potomac River | Cumberland, Md. | 875 | 86 | 21 | 19,900 | 14.0 | 4.44 | 10 | 0.72 |
| Passage Creek | Buckton, Va. | 87 | 32 | 31 | 4,800 | 5.1 | 4.37 | 1 | 0.20 |
| North River | Rockbridge Baths, Va. | 329 | 45 | 27 | 7,700 | 14.4 | 4.28 | 22 | 1.52 |
| South Fork, Shenandoah River | Front Royal, Va. | 1,635 | 146 | 7.5 | 30,400 | 12.1 | 4.27 | 7 | 0.58 |
| Sandy River | Danville, Va. | 113 | 21 | 21 | 5,600 | 12.9 | 4.10 | 11 | 0.86 |
| Roanoke River | Brookneal, Va. | 2,420 | 154 | 6.1 | 32,600 | 13.7 | 3.99 | 14 | 1.02 |
| Cow Pasture River | Clifton Forge, Va. | 456 | 69 | 13.1 | 8,700 | 8.5 | 3.94 | 16 | 1.88 |
| Appomattox River | Petersburg, Va. | 1,335 | 122 | 2.9 | 7,200 | 16.0 | 3.88 | 33 | 2.06 |
| Dan River | South Boston, Va. | 2,730 | 179 | 5.1 | 21,100 | 13.8 | 3.84 | 27 | 1.95 |
| Monocacy River | Frederick, Md. | 817 | 62 | 5.9 | 13,500 | 13.5 | 3.78 | 31 | 2.29 |
| North River | Lexington, Va. | 487 | 58 | 21 | 11,000 | 15.2 | 3.63 | 16 | 1.05 |
| South Branch, Potomac River | Petersburg, W. Va. | 642 | 60 | 24 | 14,500 | 13.9 | 3.56 | 8 | 0.57 |
| Falling River | Brookneal, Va. | 228 | 23 | 11 | 6,600 | 6.7 | 3.48 | 10 | 1.49 |
| North Fork, Shenandoah River | Cootes Store, Va. | 215 | 26 | 44 | 14,500 | 5.0 | 3.45 | 2 | 0.40 |
| Wills Creek | Cumberland, Md. | 247 | 35 | 40 | 11,100 | 13.6 | 3.43 | 3 | 0.22 |
| Roanoke River | Toshes, Va. | 1,020 | 96 | 9.2 | 21,000 | 13.6 | 3.33 | 8 | 0.59 |
| Mayo River | Price, N. C. | 260 | 38 | 20 | 9,200 | 12.0 | 3.26 | 6 | 0.50 |
| Pig River | Toshes, Va. | 394 | 72 | 8.7 | 8,600 | 12.7 | 3.26 | 7 | 0.55 |
| Otter River | Evington, Va. | 325 | 34 | 18 | 8,600 | 6.5 | 3.21 | 7 | 1.07 |
| Blackwater River | Union Hall, Va. | 208 | 46 | 14 | 6,200 | 13.6 | 3.18 | 9 | 0.66 |
| James River | Cartersville, Va. | 6,242 | 285 | 4.6 | 58,100 | 43.3 | 3.17 | 72 | 1.66 |
| James River | Lick Run, Va. | 1,369 | 78 | 17 | 21,700 | 14.5 | 3.08 | 21 | 1.45 |
| Hyco River | Omega, Va. | 338 | .... | .... | 2,400 | 8.6 | 3.02 | 38 | 4.44 |
| Roanoke River | Niagara, Va. | 511 | 53 | 16 | 11,600 | 16.8 | 3.02 | 16 | 0.95 |
| South Fork, Shenandoah River | Lynnwood, Va. | 1,076 | 63 | 12 | 28,700 | 12.1 | 2.80 | 6 | 0.49 |
| Slate River | Arvonia, Va | 235 | 34 | 9.0 | 4,400 | 6.1 | 2.79 | 8 | 1.31 |
| Roanoke River | Roanoke, Va. | 388 | 48 | 17 | 10,500 | 6.0 | 2.78 | 3 | 0.50 |
| South Fork, South Branch, Potomac River | Moorefield, W. Va. | 283 | 54 | 25 | 8,250 | 3.3 | 2.78 | 3 | 0.91 |
| North Branch, Potomac River | Bloomington, Md. | 287 | 49 | 45 | 8,100 | 13.8 | 2.76 | 10 | 0.72 |
| Goose Creek | Huddleston, Va. | 187 | 33 | 14 | 6,400 | 12.7 | 2.68 | 10 | 0.79 |
| Savage River | Bloomington, Md. | 115 | 26 | 65 | 4,560 | 13.7 | 2.63 | 6 | 0.44 |
| Jackson River | Falling Spring, Va. | 409 | 47 | 22 | 9,500 | 8.5 | 2.62 | 9 | 1.06 |
| Georges Creek | Franklin, Md. | 72.4 | 18 | 78 | 3,320 | 13.6 | 2.56 | 4 | 0.29 |
| Smiths River | Martinsville, Va. | 374 | 48 | 13 | 15,500 | 13.6 | 2.52 | 8 | 0.59 |
| North River | Burketown, Va. | 375 | 18 | 37 | 17,500 | 3.9 | 2.46 | 3 | 0.77 |
| Dan River | Francisco, N. C. | 124 | 43 | 35 | 5,100 | 12.0 | 2.44 | 5 | 0.42 |
| Antietam Creek | Sharpsburg, Md. | 281 | 49 | 11 | 2,830 | 9.9 | 2.43 | 8 | 0.81 |
| South River | Waynesboro, Va. | 144 | 24 | 17 | 4,900 | 14.3 | 2.42[f] | 8 | 0.56 |
| Craig Creek | Parr, Va. | 331 | .... | .... | 6,900 | 5.8 | 2.20 | 12 | 2.06 |
| Conococheague Creek | Fairview, Md. | 494 | .... | .... | 7,600 | 9.7 | 2.14 | 10 | 1.04 |
| Middle River | Grottoes, Va. | 360 | 54 | 14 | 12,600 | 4.2 | 1.67 | 3 | 0.70 |
| Hardware River | Scottsville, Va. | 116 | 22 | 18 | 2,600 | 4.4 | 1.33 | 4 | 0.92 |
| Seneca Creek | Dawsonville, Md. | 101 | 17 | 17 | 5,560 | 12.8 | 1.17 | 1 | 0.08 |

[a] Total length of longest low water flow path. [b] Uniform slope of profile along longest flow path which would have same area under profile and above low water elevation at gage. [c] Peak discharge for 1.0-in. runoff generated uniformly in 6 hours. [d] Discharges of actual floods in analyzed record in ratio to unit flood peak. [e] Number of floods analyzed in excess of 0.9 unit peak. [f] Prior to record analyzed, flood of September 30, 1870 (relatively the largest known flood at any of the stations analyzed), reached a flow estimated as 8.0 times unit flood peak.

Probable Average Flood Interval, in Years
Ratio of Flood Peak Discharges to Unit Flood Peak Discharge
Average Percentage Chance of Occurrence per Year
Probable Average Flood Interval, in Years
Average Percentage Chance of Occurrence per Year
LEGEND
Stations in Potomac River Basin
Stations in James River Basin
Stations in Roanoke River Basin
(a) 834 OCCURRENCES TREATED AS A SINGLE RECORD
(b) 834 OCCURRENCES GROUPED BY SIZE OF DRAINAGE AREAS
70 to 400 Sq Mi
400 to 2500 Sq Mi
2500 to 10000 Sq Mi
70 to 10000 Sq Mi
Drainage Areas

FIG. 19.—FLOOD FREQUENCIES, POTOMAC, JAMES, AND ROANOKE RIVERS, FOR DRAINAGE AREAS OF FROM 70 SQ MILES TO 10,000 SQ MILES

A spread in the curves results. This spread is no greater than that commonly experienced in the analysis of flood records for a single station. Furthermore, relative size in the rare frequency range does not increase or decrease consistently with increasing area, although a consistent trend does appear in the range of 1-yr to 5-yr floods. Until a contrary theory is demonstrated conclusively, the influence of watershed area would be regarded as negligible, except in the range of very frequent floods where flows would tend to be more nearly proportional to watershed area.

The accuracy of the similar assumptions with regard to other watershed characteristics might be checked similarly. An illustrative application to the aforementioned data is prohibited by the labor of evaluating the factors. However, since the authors have already evaluated more factors than they were able to report in their flood formulas, they may find re-examination of their data feasible.

Exact determination of the peak of the unit hydrograph of a single stream is not as simple as much published material might appear to indicate. As the authors have shown, different investigators using the same data will not agree, and investigators using different data from the same stream or different methods of analysis may reach quite different conclusions. Much of the variation is random in character and therefore tends to average in the analysis of a large number of streams.

The results of a general analysis like the foregoing, which is applied to a single stream, would embody any errors that might be involved in determining that particular unit hydrograph peak, and should be checked accordingly. If the record of the single stream were long enough to establish a reasonably accurate 10-yr flood, the relative size of rarer floods in proportion to the decennial flood might be used instead of the proportion of rarer floods to the unit flood. This would be tantamount to revision of the unit hydrograph peak by such an amount as would make the actual 10-yr flood and the computed 10-yr flood agree. Judgment would decide whether such an adjustment should be made. The writer knows no acceptable criterion for determining the amount of variation from mean flood experience which can occur without a stream being conclusively indicated as an exception to a stated probability law. Some records of 60 to 100 years show 10-yr periods deviating by 400% from the mean flood experience of the entire period; that is, four 10-yr floods when one would be normal for 10 years.

Some adjustment of the type mentioned in the preceding paragraphs is necessary when comparing conclusions of different investigators. Table 5 shows a comparison of frequency curves for the Middle Branch of Westfield River at Goss Heights; that is, Fig. 12 is compared with data that would develop from the frequency study of Fig. 19(*a*) (70 to 10,000 sq miles) adjusted to make the 10-yr values agree.

The agreement in lines 3 and 4, Table 5, is much closer than the writer had expected. However, this illustrates the fact that conclusions of many independent investigators may agree much more nearly than the techniques or assumptions that the investigators might use to reach the conclusions. For this reason the writer does not like to argue about the validity of many individual hydrological concepts and premises. Few conclusions depend on one

such concept; most depend on the combination of many. Divergent theories about one premise are counterbalanced by opinionated variations on the opposite side. The writer belongs to a school of thought which denies the validity of unit hydrograph peak variation with volume of runoff and, without

TABLE 5.—COMPARISON OF FREQUENCY CURVES FOR THE MIDDLE BRANCH OF THE WESTFIELD RIVER

| No. | Description | AVERAGE RECURRENCE INTERVAL, IN YEARS | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 5 | 10 | 20 | 50 | 100 | 1,000 |
| | Ratio of Flood Discharge; from Fig. 19(*a*): | | | | | | | | |
| 1 | To unit flood | 0.95 | 1.40 | 2.08 | 2.68 | 3.28 | 4.10 | 4.75 | 6.95 |
| 2 | To decennial flood | 0.34 | 0.52 | 0.78 | 1.00 | 1.22 | 1.53 | 1.77 | 2.60 |
| | Discharge, in Cubic Feet per Second: | | | | | | | | |
| 3 | From Fig. 19(*a*) | 2,100 | 3,100 | 4,700 | 6,000 | 7,300 | 9,200 | 11,000 | 16,000 |
| 4 | From Fig. 12 | 2,600 | 3,400 | 4,800 | 6,000 | 7,400 | 9,700 | 12,000 | 20,000 |

intending to criticize the authors' excellent analyses, he attributes such indications to an inherent fallacy in a very commonly used method for the derivation of unit graphs and to the common neglect of nonuniformity of runoff distribution in the derivation and use of unit graphs. On the other hand, his techniques for analyzing the nonuniformity of areal distribution and runoff factors are extensive. In any actual use, both techniques would be used together and seasoned with the advocates' judgment and experience. The results would be compared with a record. Presumably, neither the authors' nor the writer's analyses would agree with the record exactly, although both might be acceptably close.

Hydrology and the hydraulics of natural stream flow are complex, interrelated phenomena, too involved for practical use or exact understanding. As a result, most analyses are oversimplified explanations of an observed phenomenon in terms of a limited number of mentally conceived rational premises. The exact truths of the premises cannot be demonstrated, since only a combined result can be observed. Thus, the combination of rational concepts is the yardstick by which an individual calibrates his own judgment.

Papers of this kind present a yardstick for comparisons with other similar standards. If all yardsticks are the correct length, it is frequently presumptuous to argue the merit of the pieces. Sometimes, however, through such argument, one discovers that what he thought was an essential piece of his yardstick is either a useless piece hanging on the side, or an unnecessary piece that has been put in at one end and taken off at the other and thus adds only dead weight to the whole.

If this modified analysis is as appropriate when applied to New England stream flow records as when developed from records elsewhere, it will indicate that the same conclusions could have been reached without use of rainfall records or conversions of rainfall to runoff. It would then raise the question: "Are the premises which deal with these elements really part of the yardstick or are they computationally just extra pieces which went into the derivation at one place and out at another without really influencing the answer?"

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# ECONOMICAL CANAL CROSS SECTIONS

## Discussion

BY D. C. MCCONAUGHY, AND C. J. POSEY

D. C. MCCONAUGHY,[7] ESQ.[7a]—For the numerical example chosen, the author finds that the parabola and the semicubical parabola are the most economical shapes. However, the slopes at the water surface for these sections, given in Table 2, vary from 0.86 to 1.20. These slopes, in some cases, would not be practicable for the excavation. The slopes at the water surface for the trapezoidal section and for the section with straight sides and curved bottom vary, respectively, from 1.25 to 1.42 and from 1.20 to 1.40, which ranges are more desirable for some soils. It becomes of interest, therefore, to compare the cost of the most economical section with that of the sections more nearly resembling those ordinarily used. If the term $C$ in Eq. 5 is assumed the same for all sections, relative costs in the author's illustrative example are obtained from the remaining terms and are shown in Table 3. The volume of excavation is, roughly, 60 cu yd per ft, so that a small additional cost of excavation for the curved sides would offset any supposed economy due to the shape. The lining thickness is taken as the same for all sections. Whether this would be the case, assuming that the excavated slopes could be maintained at least temporarily, would depend on the judgment of the designer as to whether or not the slope could be regarded as permanent.

In Eq. 2, the stiffness factor $\sigma$ seems to be a measure of the total force tending to slide the lining down the side slopes; in other words, an increase in $\sigma$ seems to mean that there is less friction of the concrete on the sides of the excavation. This might result from a low coefficient of internal friction for the ground. In that case, the author's example shows that the section should be made deeper and the sides steeper—a rather peculiar result.

If the example chosen may be considered representative, the curved bottom with straight sides would seem nearly as cheap as the more elaborate sections, and more practicable.

NOTE.—This paper by Victor L. Streeter, Assoc. M. Am. Soc. C. E., was published in May, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1944, by V. A. Endersby; and October, 1944, by Julian Hinds.

[7] Senior Engr., U. S. Bureau of Reclamation, Denver, Colo.

[7a] Received by the Secretary September 26, 1944.

The author deserves the thanks of the profession for making available a method either of designing a section of minimum cost or of ascertaining how much the cost of any given section that may be considered desirable departs from the absolute minimum.

TABLE 3.—COST, IN DOLLARS PER FOOT, OF CANALS WITH VARIOUS CROSS SECTIONS

| Section | Type | Values of $y_c$: | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 10 | 15 | 20 | 25 |
| (*a*) Steepness Index ($\sigma$) = 0.75 | | | | | | |
| 1 | 4 | 20.57 | 24.57 | 26.57 | 28.56 | 30.55 |
| 2 | 5 | 20.40 | 24.40 | 26.40 | 28.39 | 30.39 |
| 5 | 8 | 20.33 | 24.35 | 26.35 | 28.36 | 30.37 |
| (*b*) Steepness Index ($\sigma$) = 1.50 | | | | | | |
| 1 | 4 | 20.30 | 24.19 | 26.13 | 28.07 | 30.02 |
| 2 | 5 | 20.06 | 23.90 | 25.82 | 27.75 | 29.67 |
| 5 | 8 | 19.80 | 23.58 | 25.47 | 27.37 | 29.26 |
| (*c*) Steepness Index ($\sigma$) = 2.25 | | | | | | |
| 1 | 4 | 20.03 | 23.81 | 25.70 | 27.59 | 29.48 |
| 2 | 5 | 19.71 | 23.41 | 25.26 | 27.11 | 28.96 |
| 4 | 7 | 19.38 | 23.07 | 24.91 | 26.76 | 28.60 |
| 5 | 8 | 19.44 | 23.04 | 24.84 | 26.64 | 28.44 |

C. J. POSEY,[8] Assoc. M. Am. Soc. C. E.[8a]—According to the author, concrete can be placed higher on the side slopes of a canal, without using cover forms, than would be permitted by friction alone. This seems quite reasonable, although contrary to the textbook statements that freshly poured concrete has the properties of a fluid. The extra force necessary to hold the concrete up along the slope must be a compressive resistance in the fresh concrete. For a V-shaped cross section, with sides too steep to support the concrete by friction, this force would increase uniformly to a maximum at the bottom. If the concrete were placed too high on the sides, failure would result when the accumulated compressive force in the concrete at the bottom became so great as to cause the section to bulge out of shape and allow the concrete on the sides to slip down. For the V-shaped cross section, the compressive force from one side is balanced entirely by the compressive force from the other, assuming, of course, that the lining was poured to the same height on each side.

If the section were trapezoidal, with sides steeper than the angle of friction, the failure would follow nearly the same pattern. The compressive force at the base of the slope would have to be resisted by a compressive force in the bottom lining. The intensity of this force would decrease away from the corner, due to friction on the bottom, becoming zero in the central part if the bottom were wide enough.

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[8a] Received by the Secretary September 28, 1944.

In the author's more general case, the compressive force in the concrete increases from zero at the top to a maximum at some point part way down the slope. Below this point the frictional forces exceed the component of the weight. At this point the frictional forces acting on a differential length $dl$ are equal to the component of the weight along the slope. The compressive stress in the fresh concrete at this point, a distance $y_0$ from the water surface, helps support the concrete of the upper lining, which is on too steep a slope to remain there by friction alone.

In the development of the "steepness index," the author assumes that the frictional force resisting motion is independent of the angle of the subgrade. This use of a constant value for the frictional force, as given in Eq. 1, permits a very simple integration for the maximum accumulated force, but is open to question, for it is equivalent to assuming that the frictional force is all cohesion. It is difficult to think of the concrete as being capable of sustaining any appreciable accumulated compressive force if its properties do not include internal friction as well as cohesion. The frictional force between the concrete and the subgrade must vary, at least to some extent, with the normal pressure. The author's assumption of a constant value throughout may or may not be satisfactory. Experiments simulating the action of the paving machines in depositing concrete on different slopes would be helpful in evaluating the adequacy of the steepness index.

In Eq. 5, it would seem reasonable to assume that the cost of excavation varies as the $n$th moment of the cross section about the natural ground surface, $n$ having a value near unity. For the comparison of canals of generally similar proportions, $n$ might be taken as zero (as the author has done, since the area is the 0th moment), but, in considering deeper cross sections, the increased power requirement for elevating the excavated material should be taken into account.

The author is to be commended for his pioneering effort in a field which is destined to become more active as the labor of canal digging and paving is more completely mechanized. He is undoubtedly correct in suggesting that the old sharp corners, which are inefficient from both hydraulic and structural viewpoints, will be eliminated in favor of curves. It is well that the theoretical investigations keep ahead of the immediate demands of practice. Otherwise, designers might specify old-style cross sections in order to avoid the labor of computing the characteristics of a section better suited to their particular problems. On the other hand, the theoretical investigations should be based on as solid a foundation as possible. It might be important, for this problem, to examine critically the hydraulic friction formulas used, since results may be very sensitive to changes in formulas, and present confidence in the Manning and Kutter formulas may be based on data from too few types of cross sections. The writer has computed tables for the section illustrated in Fig. 5.[9] While making these computations, he wondered whether the Manning formula would give reliable results, when applied to such a cross section. No evidence either way has come to his attention.

[9] "Hydraulics of Steady Flow in Open Channels," by S. M. Woodward and C. J. Posey, John Wiley & Sons, Inc., New York, N. Y., pp. 13, 14.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# AMPLIFIED SLOPE DEFLECTION

## Discussion

BY LEON BESKIN

LEON BESKIN,[8] ASSOC. M. AM. SOC. C. E.[8a]—In so far as this paper lacks a definite sign convention, it tends to obscure, rather than to clarify, the analysis of rigid frames. For example, the writer has been unable to interpret the sign convention the author has chosen for moments.

In Fig. 2, it seems that at node A, the condition of equilibrium, by Eqs. 8*a* and 8*b*, is

$$M_{AB} = M_{AD} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

whereas, at node D, by Eq. 10,

$$M_{DO} + M_{DA} = M_{DC} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

In Eq. 13*b*, the subscript DO refers to the lower left-hand column of Fig. 2.

A widely adopted sign convention requires that positive moments rotate all nodes in the same direction, with the consequence that the sum of all moments at a node is zero. This is not satisfied by Eqs. 13. In the theory of continuous beams, moments at the two sides of a node are equal. In that case, Eqs. 13 should be written

$$M_{AD} = \pm M_{AB} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

and

$$M_{DO} - M_{DA} = \pm M_{DC} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

the sign in the right-hand quantity being plus or minus depending on whether the member is vertical or horizontal, respectively. Eqs. 13 do not satisfy this second sign convention.

The foregoing remarks apply also to Fig. 3. One may assume that the author's sign convention is: Along columns, positive moments tend to rotate nodes clockwise; and, along beams, positive moments tend to rotate nodes counterclockwise. This convention is consistent with the conditions shown

NOTE.—This paper by Ralph W. Stewart, M. Am. Soc. C. E., was published in September, 1944, *Proceedings*.

[8] Structures Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

[8a] Received by the Secretary October 3, 1944.

in Figs. 2 and 3, but inconsistent with Fig. 1, in which $FEM_A$ is shown clockwise. Also, on this basis, Eq. 7*a* is inconsistent with Eq. 8*a*.

The comments in the text following Eq. 7*b* are also erroneous:

> "It is well known to all users of the slope-deflection method that the final moment at the end of a beam for which the moments are not dominated by joint movements is less than the fixed-end moment."

The final moment can be greater or smaller than the fixed-end moment, as demonstrated by a continuous beam on three supports for which the moment at the center support has an intermediate value between the fixed-end moments calculated for each span independently—it is greater than one moment, smaller than the other.

The rule following the foregoing quotation is not clear, as its wording seems to apply to absolute values, although the succeeding sentence expresses a contrary opinion. Moreover, the moment due to the deflection may have any sign, independent of the fixed-end moments.

The end-moment formula (Eq. 7*a*), when applied to the end B, can be written either

$$M_{BA} = \mathrm{FEM_{BA}} + 2\,E\,K\left(2\,\theta_B + \theta_A - 3\,\frac{d}{l}\right) \quad \ldots\ldots\ldots (15a)$$

or

$$M_{BA} = \mathrm{FEM_{BA}} - 2\,E\,K\left(2\,\theta_B + \theta_A - 3\,\frac{d}{l}\right) \quad \ldots\ldots\ldots (15b)$$

It has been demonstrated that a minus sign should appear in the expression of $M_{BA}$ which means that Eq. 15*b* must be used. Eqs. 8*b* and 10 tend to confirm this interpretation, although their signs are generally inconsistent with those in Eqs. 7*a* and 15*b*.

The claim for simplicity resulting from the use of the amplified slope deflections is not established, as shown by the following comparison, referring to Fig. 2: Applying Eq. 7*a* to ends A and D of member AD,

$$M_{AD} - \mathrm{M_{DA}} = -\,(\theta_A - \theta_D) \quad \ldots\ldots\ldots\ldots\ldots (16)$$

Similarly, applying Eq. 7*a* to members AB and CD, and expressing the node equilibriums,

$$-\,M_{AB} = 2\,(3\,\theta_A) = 6\,\theta_A = M_{AD} \quad \ldots\ldots\ldots\ldots (17a)$$

and

$$-\,M_{DC} - 100 = 6\,\theta_D - 100 = M_{DA} \quad \ldots\ldots\ldots\ldots (17b)$$

Adding Eqs. 17*a* and 17*b*, and noting from equilibrium considerations that the total moment in member AD is equal to 100, the resulting equation is identical with Eq. 11. By equating Eqs. 17*a* and 17*b* and substituting the difference in Eq. 16, Eq. 11 becomes identical with Eq. 12. The only difference as compared with the author's calculation is that the foregoing has taken care of sign conventions—namely, all moments that tend to rotate nodes counterclockwise are positive.

Because he eliminates moments, the author asserts that he uses two unknowns. Actually, there are five unknowns, the three additional unknown moments being eliminated rapidly. It is possible to present the calculation with one apparent unknown $M_{DA}$. Thus, by equilibrium conditions and by deformation considerations:

$$-\frac{\theta_A}{6} = M_{AB} = -M_{AD} = -100 + M_{DA} \quad \text{(18a)}$$

and

$$-\frac{\theta_D}{6} = M_{DC} = -M_{DA} - 100 \quad \text{(18b)}$$

Substituting in Eq. 16 all quantities expressed as functions of $M_{DA}$, $100 - M_{DA} - M_{DA} + \frac{100 - M_{DA}}{6} - \frac{M_{DA} + 100}{6} = 0$; and $\frac{14}{6} M_{DA} = 100$. This development is even more rapid than that proposed by the author. The author's claim that fewer equations are used and half of the number of operations are necessary, as compared with conventional methods, is doubtful. In the case of the unsymmetrical framework, with a hinge at point D, a straightforward method applying strain energy requires the use of only three unknowns—the shear in the vertical member OD, the shear in member CD, and the axial load in member CD.

Of minor importance are the unjustifiable introduction of the symbol $\Delta$ from a technique that bears no remote relation to stress analysis,[2] and the author's adoption of the inaccurate term "tapered beams" to describe "unsymmetrical beams with variable moments of inertia."

In conclusion, it seems to the writer that slope-deflection methods do not require clarification. The author's effort to clarify them is imperfect to the extent that his lack of a precise sign convention has introduced unnecessary confusion.

[2] "The Railway Transition Spiral," by A. N. Talbot, McGraw-Hill Book Co., Inc., New York, N. Y. (6th Ed., 1927, out of print).